Aspects of confinement in QCD from lattice simulations

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ITP Heidelberg

Seminar *'Quantum Dynamics, Non-equilibrium Physics and Simulational Methods'*
November 22, 2010

Talk mainly based on

- J. M. Pawlowski, DS, I.-O. Stamatescu:
  Lattice Landau gauge with stochastic quantisation
  0911.4921 [hep-lat], Nucl. Phys. B 830:291, 2010

- A. Maas, J. M. Pawlowski, DS, A. Sternbeck, L. von Smekal:
  Strong-coupling study of the Gribov ambiguity in lattice Landau gauge
  0912.4203 [hep-lat], Eur. Phys. J. C 68:183, 2010

- J. M. Pawlowski, DS, A. Sternbeck, L. von Smekal:
  work in preparation (on free b.c.)

- A. Maas, J. M. Pawlowski, DS, L. von Smekal:
  work in preparation (on propagators at $T > 0$)

- G. Aarts, S. Hands, J. M. Pawlowski, D. Sexty, DS, I.-O. Stamatescu:
  unpublished work (on complex stochastic quantization)

Outline

1. Introduction
2. Confinement mechanism and infrared propagators
   - Stochastic quantization for gauge fixing
   - Strong-coupling limit in $d = 2, 3, 4$
   - Gribov ambiguity in the strong-coupling limit
   - Free boundary conditions
3. Deconfinement phase transition at $T > 0$
4. Summary

Outline

Structure

Main subject

- Confinement mechanism in Landau gauge
  - Relation to infrared propagators
  - Gauge fixing problem

Two 'spin-off' projects

- Deconfinement phase transition at $T > 0$ from the gluon propagator
- Sign problem for fermions at $\mu > 0$ with complex stochastic quantization *($\leftarrow$ no time in this talk ...)*

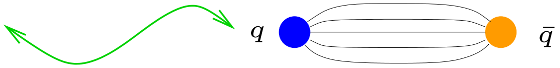
Now: introduction to the main subject ...

Confinement and infrared propagators

Signatures of confinement

- Confining quark-antiquark potential (gauge-invariant)
  - demands an explanation
- Infrared behavior of gluon & ghost propagator (gauge-dependent)
  - may provide an explanation
    - ★ Gribov–Zwanziger scenario
      – related to confinement via topological defects
    - ★ Kugo–Ojima scenario
  - here: Landau gauge, $\partial_\mu A_\mu = 0$

$$\langle AA \rangle$$
$$\langle c\bar{c} \rangle$$



Gluon & ghost propagator

Basic task

Gauge fixing

$\downarrow$

extract infrared Yang–Mills propagators

Question

Is the IR behavior of

- scaling type or
- decoupling type?

Gluon & ghost propagator

Landau gauge, $\partial_\mu A_\mu = 0$

- Gluon propagator:

$$\langle AA \rangle \propto \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D_{\mathrm{gl}}(q^2)$$

- Ghost propagator:

$$\langle c\bar{c} \rangle \propto -\delta^{ab} D_{\mathrm{gh}}(q^2)$$

- Dressing function $\equiv q^2 \cdot D_{\mathrm{gl/gh}}(q^2)$

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  ... related to Faddeev–Popov operator

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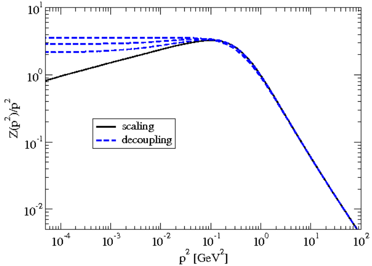
IR propagators: continuum solutions

in Landau gauge

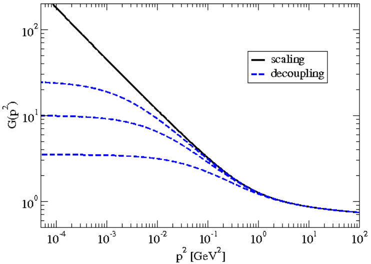
One-parameter family of solutions from FRG & DSE

(figs. from Fischer/Maas/Pawlowski '08)

Gluon propagator D_{gl}



Ghost dressing function $q^2 D_{\mathrm{gh}}$



Infrared exponents

$$\lim_{q^2 \to 0} D_{\mathrm{gl/gh}}(q^2) \propto \frac{1}{(q^2)^{\kappa_{A/C}+1}}$$

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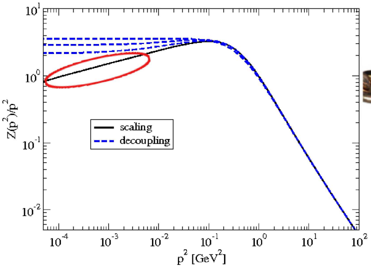

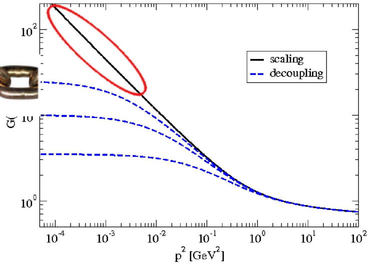

Infrared exponents (von Smekal et al. '97, Lerche/von Smekal '02, Zwanziger '01, Pawlowski et al. '03)

$$\lim_{q^2 \to 0} D_{\mathrm{gl/gh}}(q^2) \propto \frac{1}{(q^2)^{\kappa_{A/C}+1}}$$

if scaling: $\kappa_A = -2 \underbrace{\kappa_C}_{=:\kappa} + \dfrac{d-4}{2}$

IR gluon & ghost propagator

Confinement mechanism ⇒ IR behavior

Confinement scenarios (Gribov–Zwanziger, Kugo–Ojima) $\xrightarrow[\text{global BRST}]{}$ infrared scaling

Problem

- Lattice results: No scaling – rather decoupling ($d = 3,\ 4$)

IR behavior ⇒ confinement

- Scaling solution and decoupling solutions are confining

  (Braun/Gies/Pawlowski '07)
- Both violate positivity of the gluon propagator
- At issue:
  - Confinement mechanism
  - Global BRST invariance scaling: ✓ decoupling: ✗

IR gluon & ghost propagator

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Gribov problem

A glimpse at configuration space

- Gauge fixing:
  $\partial_\mu A_\mu(x) = 0 \rightsquigarrow$ hypersurface
- Different Gribov regions
  on gauge fixing surface
  - distinguished by sign of
    $\det(-\partial\mathcal{D}) = \prod_n \lambda_n$

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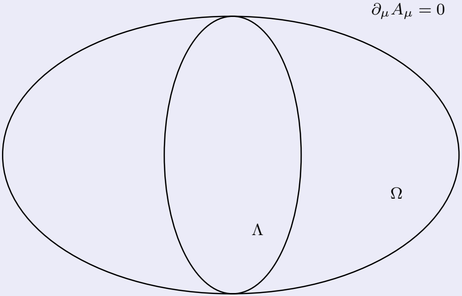
Gribov problem

Relevant regions in configuration space



- G.f. $\rightsquigarrow -\partial\mathcal{D} > 0$: 1st Gribov region Ω
- Still multiple gauge copies (Gribov '78, Singer '78)

  Gribov ambiguity
- Unique copy: Fundamental modular region Λ

  NP-hard optimization problem

Gribov problem

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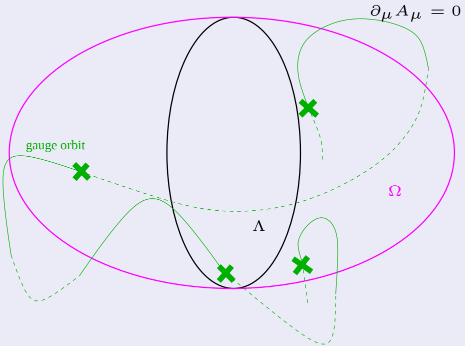

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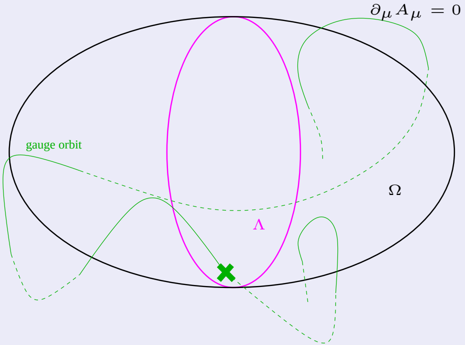

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Configuration space & gauge fixing

Motivation

- Why a non-standard gauge fixing algorithm?
  - Hope: Different sampling of configuration space.
  - Ideally, avoid ...
    - ★ possible bias (e. g. against $\partial\Omega$ or Λ)
    - ★ breaking of global BRST – remedy: topological gauge fixing (von Smekal et al. '07, '08) (not here)

Previous lattice results

- Standard lattice g. f. $\rightsquigarrow$ decoupling ($d = 3,\ 4$)
- Global maximization of g. f. functional (approach Λ? only attempted) $\rightsquigarrow$ quantitative effect, still decoupling (Bornyakov et al. '08, Bogolubsky et al. '08)
- Here: different gauge fixing methods

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Gauge fixing methods

Different methods employed here

1. Stochastic gauge fixing $\beta \equiv \frac{2N_c}{g^2} > 0$
2. Standard gauge fixing here: $\beta = 0$
3. 'max-B' gauge (non-perturbative completion of Landau gauge) here: $\beta = 0$
4. Change of boundary conditions

	g.f. per copy std.	g.f. per copy non-std.
first copy	2.	1., 4.
best copy	3.	

- here: 'best' $\hat{=}$ IR max. ghost
  - alt.: global max. of g.f. functional ($\rightsquigarrow$ 10% effect on ghost)

(Bornyakov et al. '08, Bogolubsky et al. '08)

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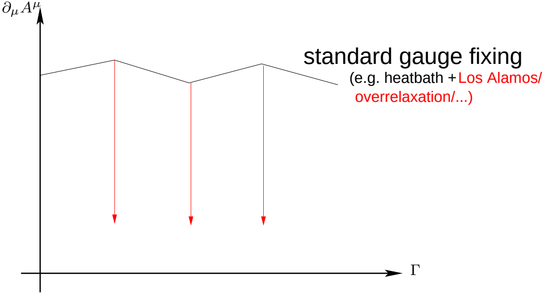
Stochastic gauge fixing: Intuitive picture



(cp. Nakamura/Saito/Sakai '04)

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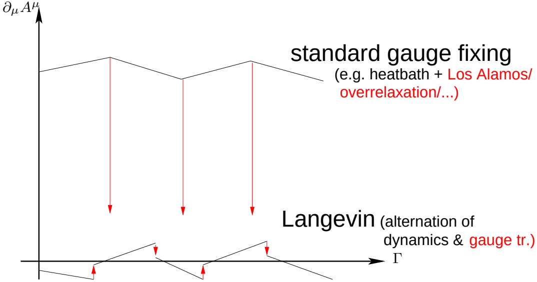


Langevin eq. incl. gauge force (Zwanziger '81)

more local than standard g. f.

$\Rightarrow$ possibly different sampling of config' space

(cp. Nakamura/Saito/Sakai '04)

Langevin dynamics

(stochastic quantization: Parisi/Wu '81, Batrouni et al. '85, Damgaard/Hüffel '87)

- Langevin equation in fictitious time θ

$$dA = (-\nabla S + \eta) d\theta$$

$\eta =$ Gaussian white noise

$$\xrightarrow[\text{equilibrium}]{} \quad \varrho[A] \propto e^{-S[A]}$$

- Correct convergence?
  - $S \in \mathbb{R}$: ✓
  - $S \in \mathbb{C}$: caution required

    possibility of
    - runaway trajectories
    - wrong convergence

    but maybe useful where other methods fail!

    ★ $\rightarrow$ sign problem ...

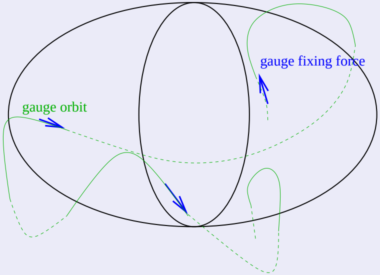
Langevin dynamics with gauge fixing (continuum)

- Supplement Langevin equation by gauge force (Zwanziger '81)

$$dA = (-\nabla S + \mathcal{D}v + \eta)d\theta$$

$\eta =$ Gaussian white noise $\quad v := \partial_\mu A_\mu$

$$\xrightarrow[\text{equilibrium}]{} \quad \varrho[A] \propto e^{-S[A]} \quad \& \quad \partial_\mu A_\mu = 0$$



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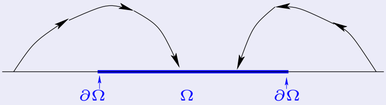
- Inside Ω: restoring towards gauge fixing surface

outside Ω: repulsive directions

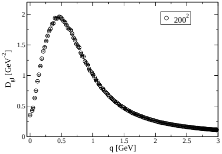
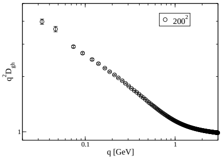
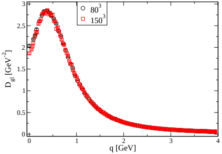
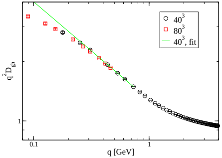
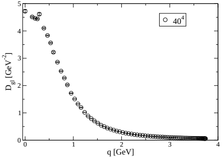
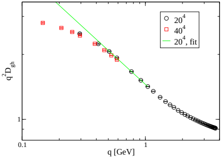


Lattice implementation

- gauge transformations of link variables $U_\mu(x)$ alternating with dynamical updates

- here: quenched $SU(2)$
- $N_c = 3$ (Bogolubsky et al. '07, Cucchieri et al. '07, Oliveira et al. '07, Sternbeck et al. '07) or dyn. fermions (Ilgenfritz et al. '06) $\Big\}$ not crucial

Propagators from stochastic gauge fixing (Pawlowski/DS/Stamatescu '09)

	gluon prop.	ghost d.f.	scaling
$d = 2$			✓
$d = 3$			✗
$d = 4$			✗

Propagators from stochastic gauge fixing

IR exponent κ: Scaling solution vs. lattice measurement

d	scaling prediction	κ_Z ghost exp. from gluon	κ ghost exp. from ghost	
2	0.2	0.19	0.17 (200^2)	✓
3	0.4	0.25	0.25 (40^3) / 0.2 (80^3)	×
4	0.6	0.5	0.26 (20^4) / 0.19 (40^4)	×

$\Rightarrow$ Further evidence for standard lattice scenario:
decoupling in $d = 3, 4$
– also with stochastic gauge fixing

(Agreement with previous results from standard methods, e.g.:
2D: Maas '07; 3D: Cucchieri/Mendes '03, 4D: Bogolubsky et al. '07, '08, '09, Cucchieri/Mendes '07)

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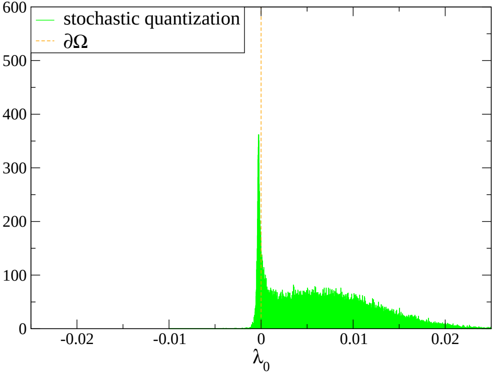
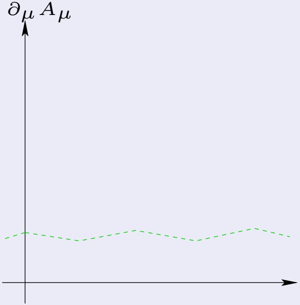
Faddeev–Popov operator spectrum



imperfect g. f.

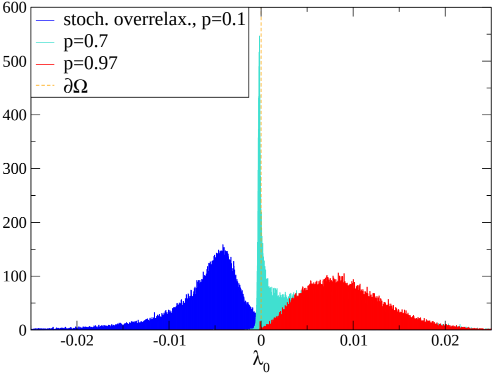


$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$
by SQ

$\Delta^a \hat{=} \partial_\mu A^a_\mu$

Peak near $\partial\Omega$
$\longrightarrow$ Gribov–Zwanziger scenario?

Faddeev–Popov operator spectrum



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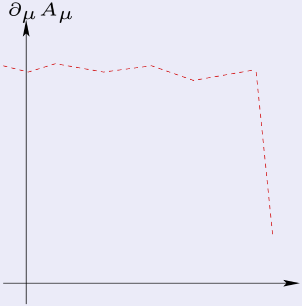
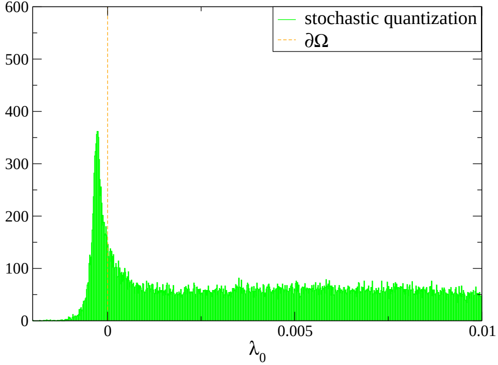


$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$
by standard g. f.

(stoch. overrel. after heatbath at different overrel. parameters)

Faddeev–Popov operator spectrum



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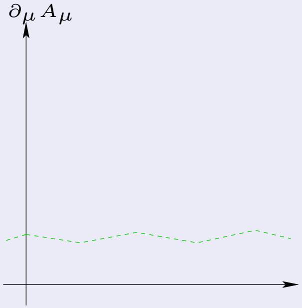

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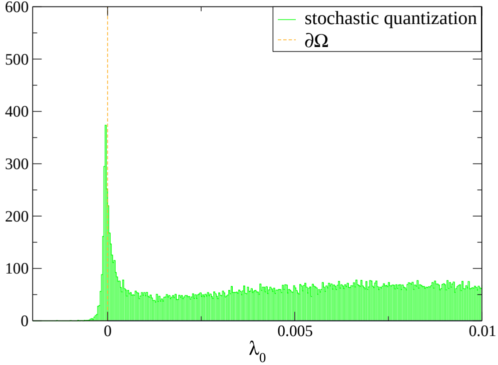
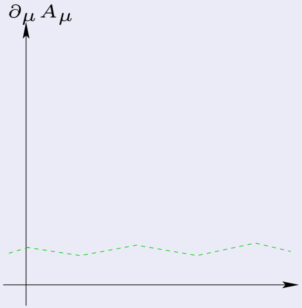
Faddeev–Popov operator spectrum

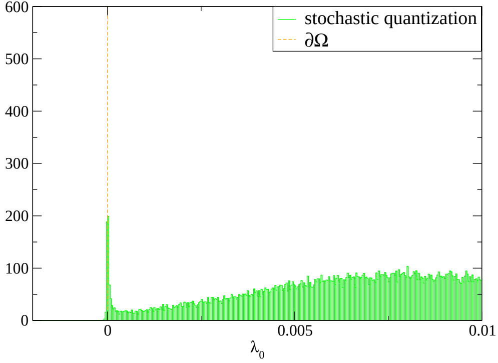


$\Delta^2 \to 0$

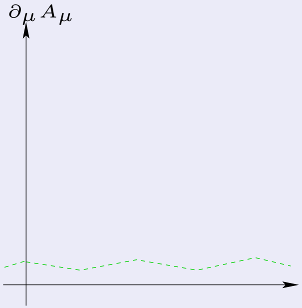


$\Delta^2 \approx 3 \cdot 10^{-5}$
by SQ

Faddeev–Popov operator spectrum



$\Delta^2 \to 0$



$\Delta^2 \approx 1.5 \cdot 10^{-6}$
by SQ

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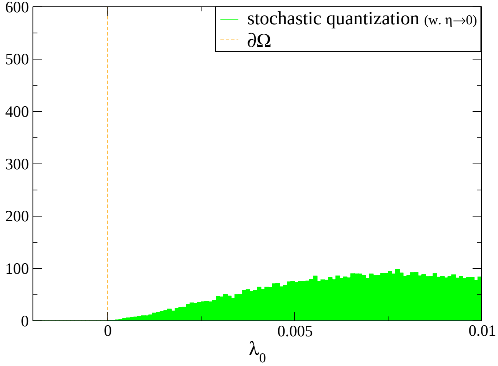
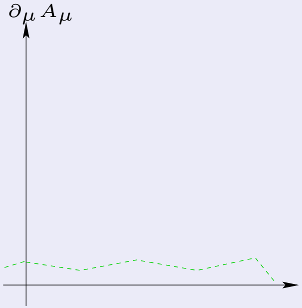


$\Delta^2 \to 0$



$\Delta^2 < 10^{-15}$
by SQ & step size $\to 0$

Peak disappears for sufficiently small Δ^2

Upshot of stochastic gauge fixing

Confirmation of standard lattice scenario
in $d = 2,\ 3,\ 4$
despite different gauge fixing algorithm

Now for some less expected findings ...

Upshot of stochastic gauge fixing

Confirmation of standard lattice scenario
in $d = 2,\ 3,\ 4$
despite different gauge fixing algorithm

Now for some less expected findings ...

Outline

Strong-coupling limit $\beta = 0$

Motivation

- $$\frac{2N_c}{g^2} = \beta \to 0 \qquad \hat{=} \qquad a \to \infty$$
  $$\Rightarrow \text{IR behavior visible at large } aq$$
- 4D: Conformal behavior (Sternbeck/von Smekal '08)
- Here: $d = 2$ & 3
- Interpretation under debate
  - position 1 (Cucchieri/Mendes '09)
    "lattice sees
    - *decoupling in $d > 2$,*
    - *scaling in $d = 2$"*
  - position 2 (Maas/Pawlowski/DS/Sternbeck/von Smekal '09)
    "issue not settled"

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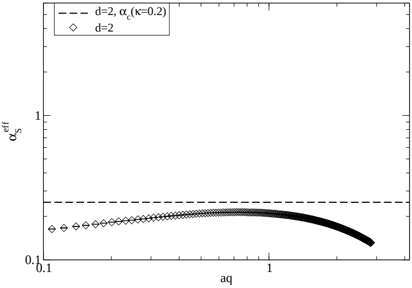
Running coupling

eff. running coupling

$$\alpha_S^{\text{eff}} = \frac{g^2}{4\pi} q^{d+2} D_{\text{gl}} D_{\text{gh}}^2$$

Running coupling, two dimensions



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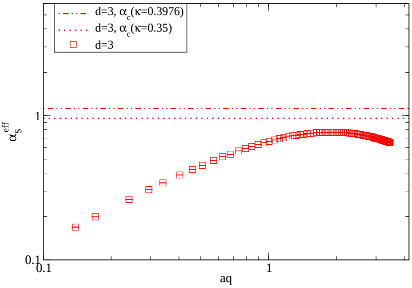
cp. with continuum predictions for α_c

(Lerche/von Smekal '02)

$$\text{peak at} \quad \left.\begin{matrix} 85\% \\ \\ \end{matrix}\right\} \text{ of } \alpha_c \text{ in } d = \begin{cases} 2 \\ \\ \end{cases}$$

Running coupling, three dimensions



cp. with continuum predictions for α_c

(Lerche/von Smekal '02)

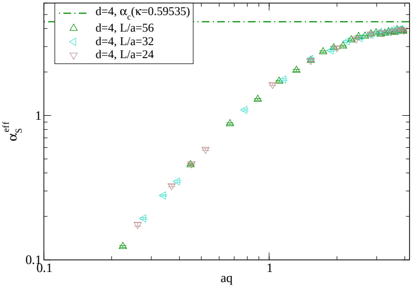
peak at 70% $\Big\}$ of α_c in $d = \Big\{ 3$

eff. running coupling

$$\alpha_S^{\text{eff}} = \frac{g^2}{4\pi} q^{d+2} D_{\text{gl}} D_{\text{gh}}^2$$

Running coupling, four dimensions

$\beta = 0$, std. g. f.



← 4d data: Sternbeck/von Smekal '08

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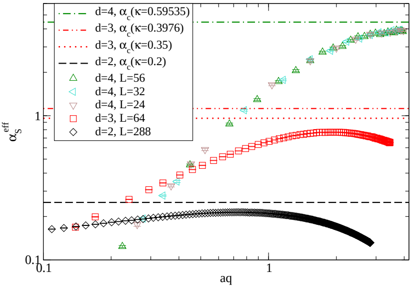
$$\text{peak at} \left.\begin{matrix} \\ 90\% \end{matrix}\right\} \text{ of } \alpha_c \text{ in } d = \begin{cases} \\ 4 \end{cases}$$

Running coupling



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cp. with continuum predictions for α_c

(Lerche/von Smekal '02)

$$\text{peak at} \begin{array}{c} 85\% \\ 70\% \\ 90\% \end{array} \Bigg\} \text{ of } \alpha_c \text{ in } d = \begin{cases} 2 \\ 3 \\ 4 \end{cases}$$

eff. running coupling

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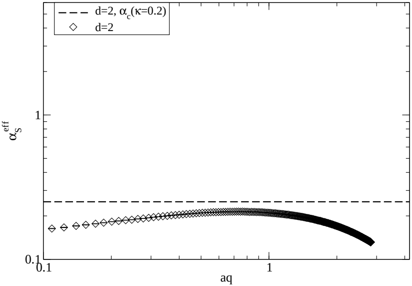
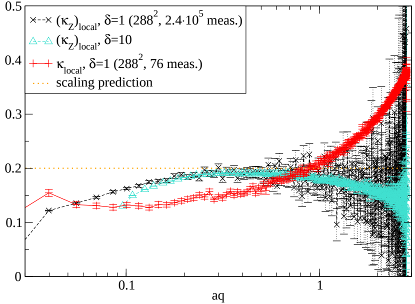
Running coupling & local exponents

$\beta = 0$, std. g. f.





$$\boxed{\alpha_S^{\mathrm{eff}} \propto \left(q^2\right)^{2(\kappa_Z-\kappa)}}$$

with κ_Z def. via scaling rel.

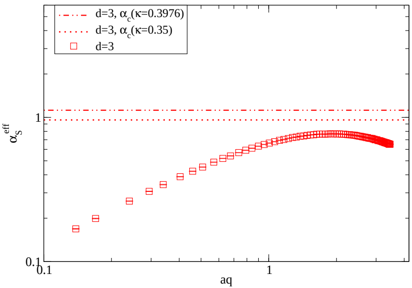
$$\kappa_A = -2\kappa_Z + \frac{d-4}{2}$$

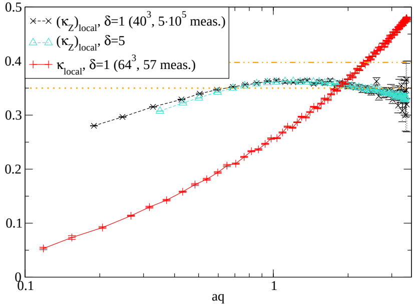
local exponents

$$\left.\begin{matrix}\kappa \\ \kappa_Z\end{matrix}\right\} = \text{ghost exp. from} \begin{cases}\text{ghost} \\ \text{gluon}\end{cases} \text{data}$$

locally from similar momenta q_i, $q_{i+\delta}$

Running coupling & local exponents





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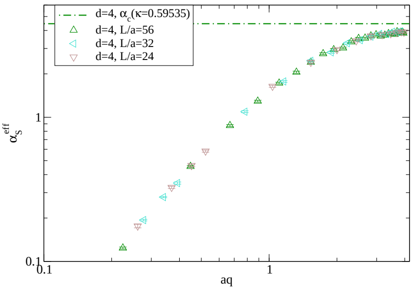

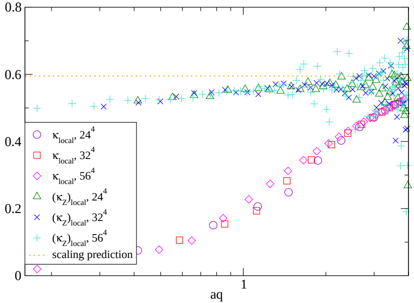

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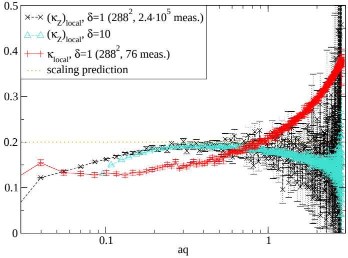
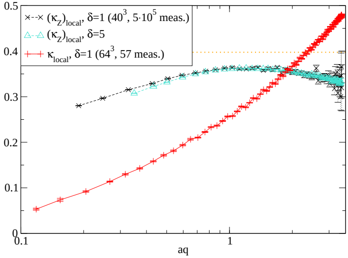
Two vs. three dimensions: Local exponents

$\beta = 0$, std. g. f.



← $d = 2$

- $d = 2$ and $d = 3$ similar at $aq > 0$
  - relation of κ and κ_Z



← $d = 3$

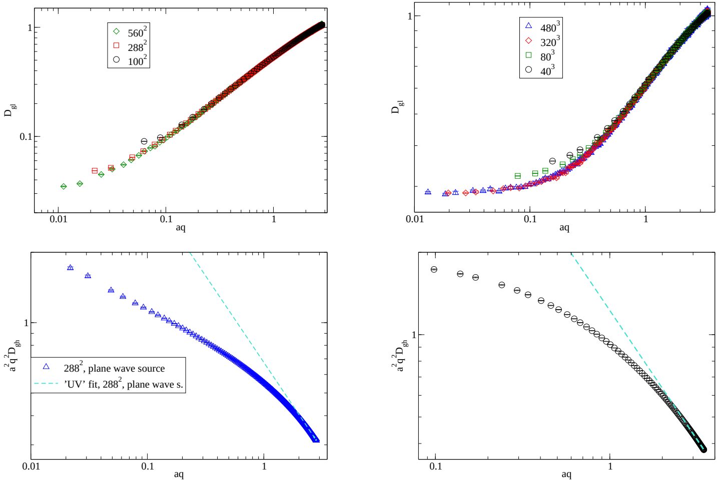
Two vs. three dimensions: Propagators

$\beta = 0$, std. g. f.

$d = 2$ $d = 3$

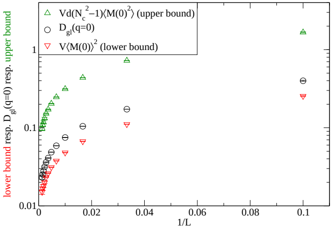
D_{gl}

$q^2 D_{\mathrm{gh}}$



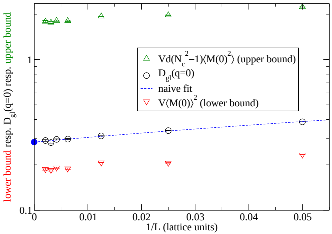
Two vs. three dimensions: $D_{\text{gl}}(q = 0)$

$\beta = 0$, std. g. f.



$\leftarrow d = 2$

- $d = 2$ and $d = 3$ similar at $aq > 0$
  - relation of κ and κ_Z
- but different at $aq = 0$
  - finite V behavior of $D_{\text{gl}}(aq = 0)$



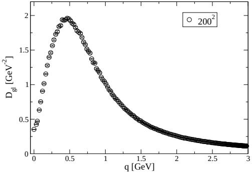
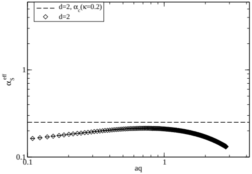
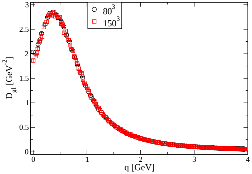
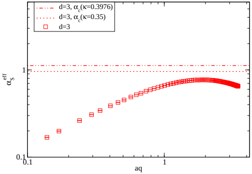
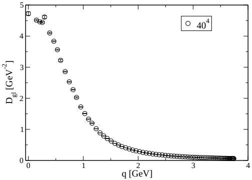
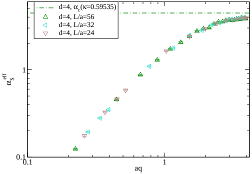
$\leftarrow d = 3$

Upshot of standard gauge fixing at $\beta = 0$

$d = 2$, 3 and 4

- no uniform scaling at all aq
- possible scaling window
  - moves towards larger aq from $d = 2$ to $d = 4$
  - same pattern at $\beta > 0$

Room for speculation ...

	gluon prop., $\beta > 0$	running coupling, $\beta = 0$
$d = 2$		
$d = 3$		
$d = 4$		

Outline

Impact of Gribov ambiguity

Basic idea of 'Landau max-B' gauge (Maas '09)

- Functional methods $\rightsquigarrow$ $\boxed{\text{one-parameter}}$ family of solutions

  (Fischer et al. '08, Boucaud et al. '08)

- Analogous lattice procedure
  -
    $$\text{parameter } B := \frac{D_{\text{gh}}(q_{\min})}{D_{\text{gh}}(Q)}$$
  - for each config'
    - ★ fix n_{copy} times to Landau gauge
    - ★ choose the copy with

      $B = \max$ (here)

- Here: first simulations at $\beta = 0$, in $d = 2, 3$

  $\beta > 0$: Maas '09 ...

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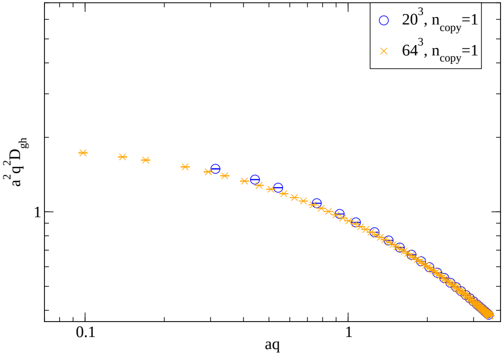
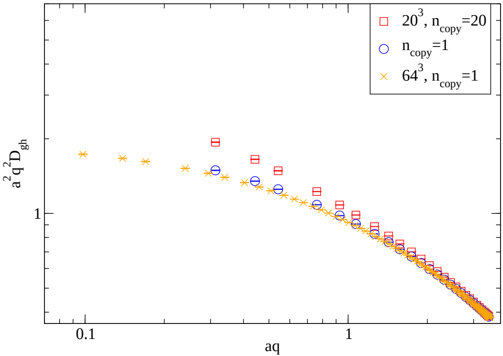


$d = 3$: ghost dressing function

- increase n_{copy} ...

Impact of Gribov ambiguity



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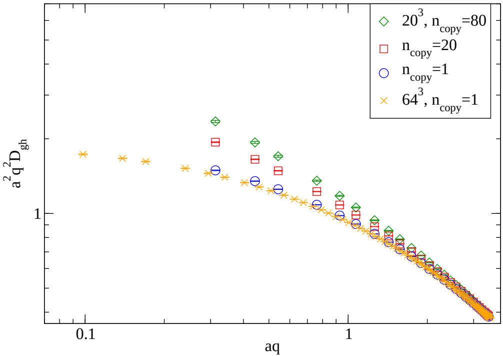

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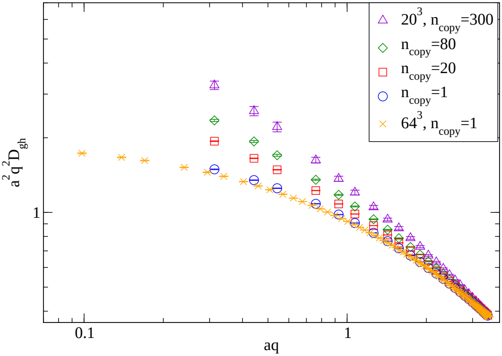

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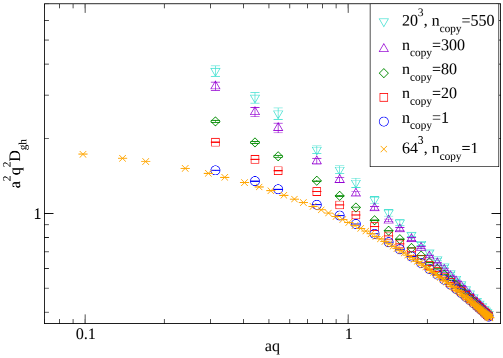
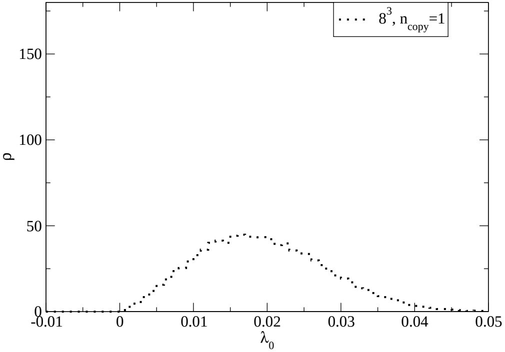


$d = 3$: ghost dressing function

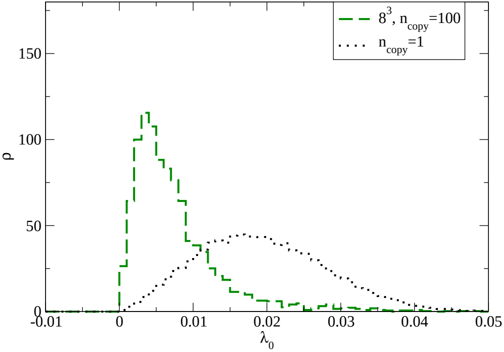
- increase n_{copy} ...
  $\rightsquigarrow$ strong effect

Impact of Gribov ambiguity



- FPO spectrum ...

Impact of Gribov ambiguity



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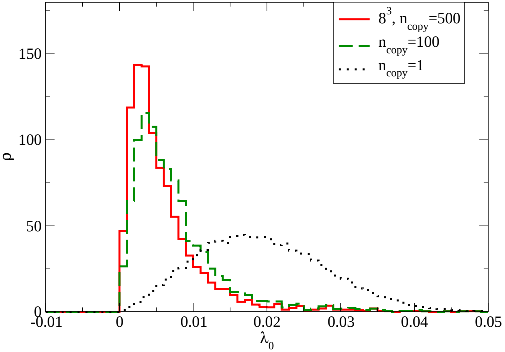
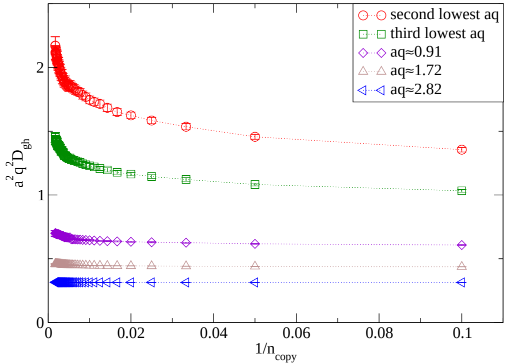
Impact of Gribov ambiguity



- FPO spectrum changes accordingly: closer to Gribov horizon

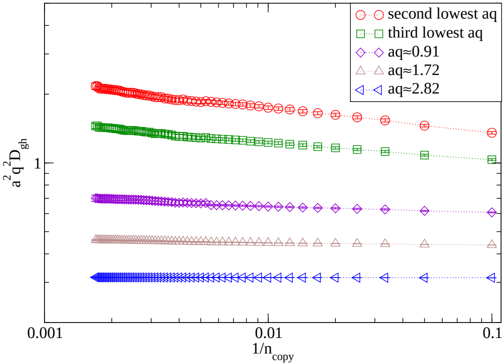
Impact of Gribov ambiguity



$d = 2$: ghost dressing function vs. n_{copy}^{-1} (48^2)

- No 'saturation' at 600 copies

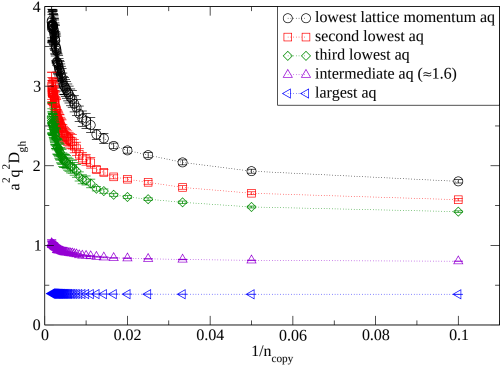
Impact of Gribov ambiguity



$d = 2$: ghost dressing function vs. n_{copy}^{-1} (48^2)

- No 'saturation' at 600 copies (log-log plot)

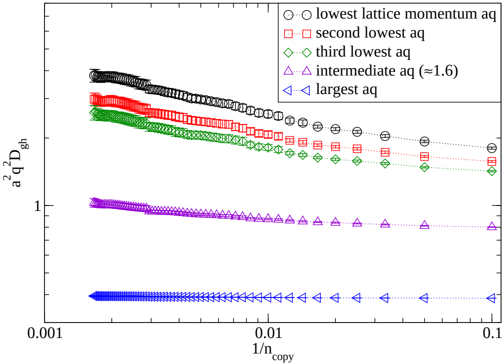
Impact of Gribov ambiguity



$d = 3$: ghost dressing function vs. n_{copy}^{-1} (20^3)

- No 'saturation' at 600 copies

Impact of Gribov ambiguity



$d = 3$: ghost dressing function vs. n_{copy}^{-1} (20^3)

- No 'saturation' at 600 copies (log-log plot)

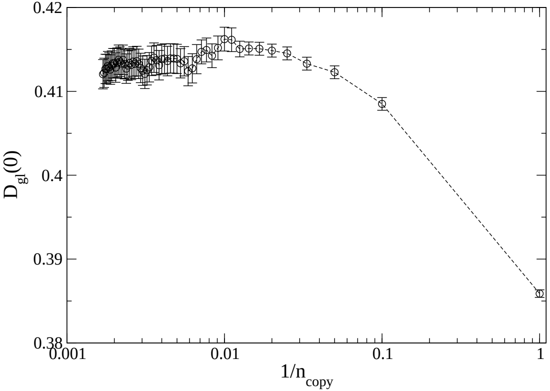
  $\Rightarrow$ Such strong effect that no $n_{\text{copy}} \to \infty$ extrapolation possible

Impact of Gribov ambiguity on α_S^{eff}

- $\alpha_S^{\text{eff}} \propto D_{\text{gh}}^2 D_{\text{gl}}$
- Gribov copy effect in max-B gauge:
  - Large impact on infrared ghost
  - Small impact on infrared gluon

here: 20^3

Impact of Gribov ambiguity on α_S^{eff}



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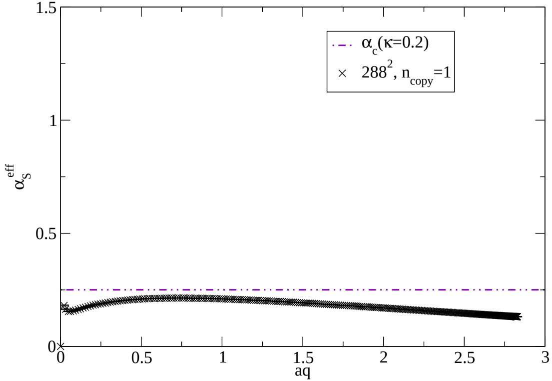
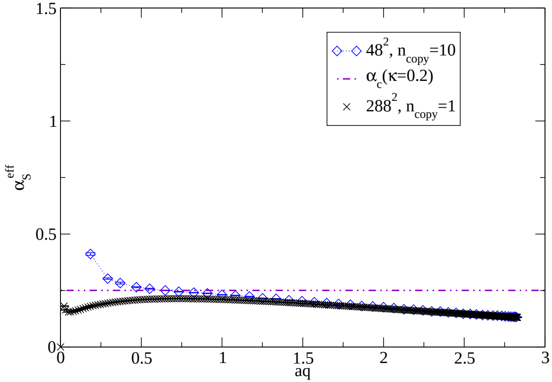


$d = 2$

Large impact
on eff. running coupling

Impact of Gribov ambiguity on α_S^{eff}



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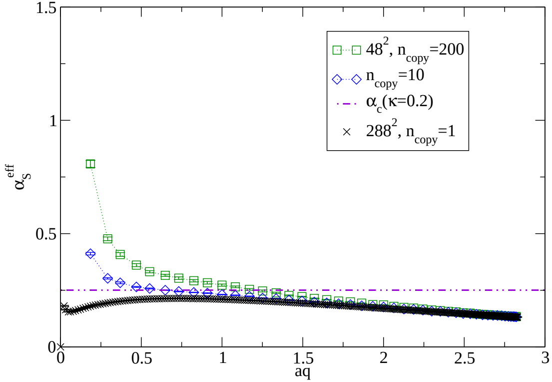

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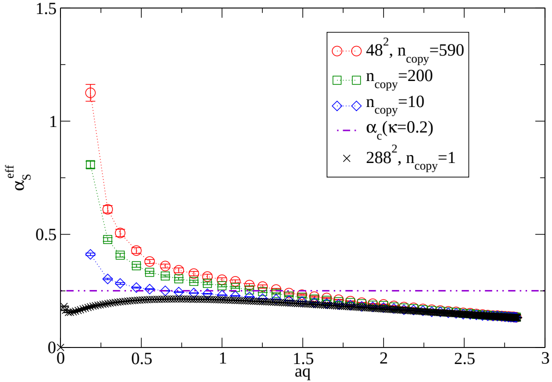

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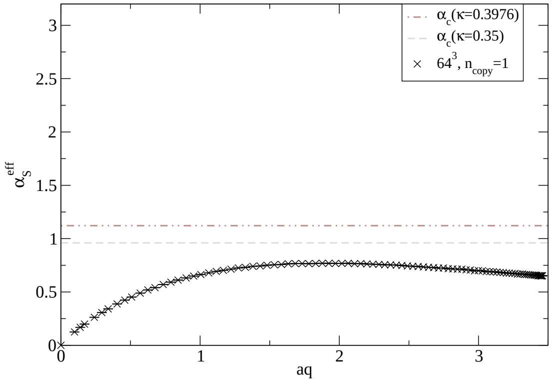
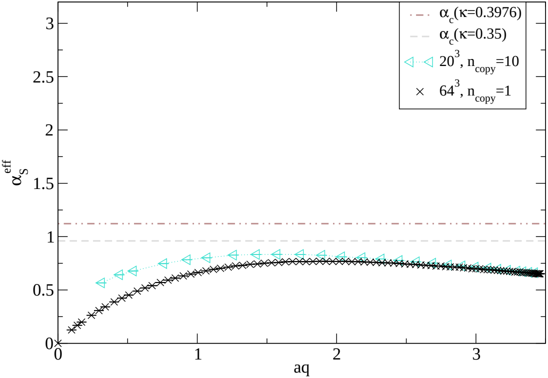


$d = 3$

Large impact
on eff. running coupling

Impact of Gribov ambiguity on α_S^{eff}



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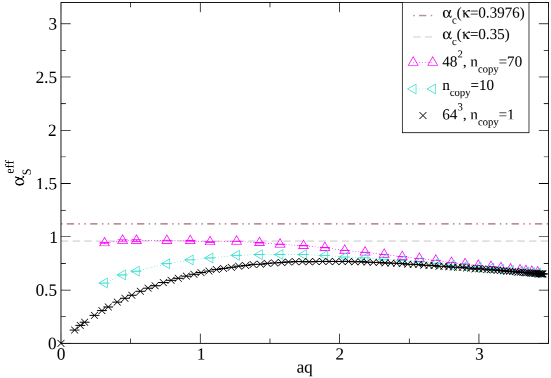

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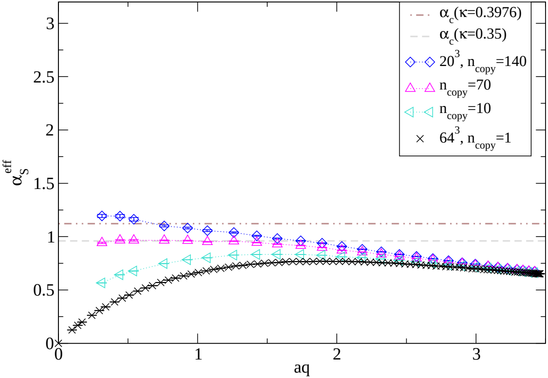

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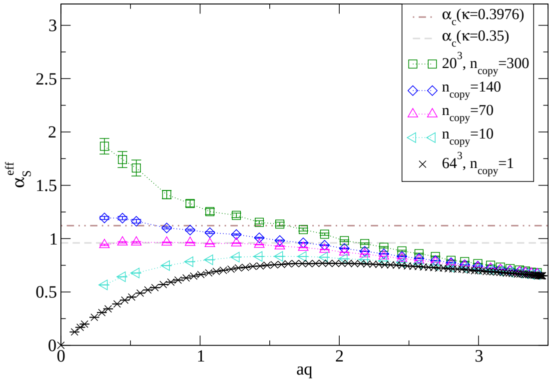

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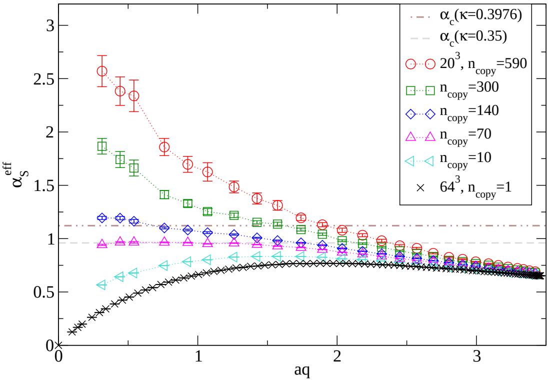
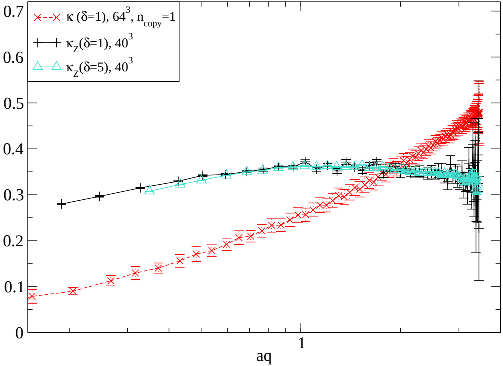


$d = 3$

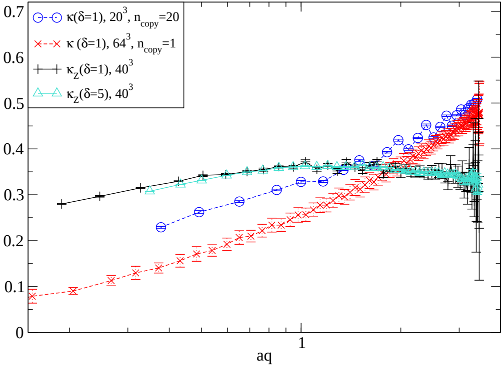
Large impact
on eff. running coupling
$\Rightarrow$ even 'over-scaling'

Impact of Gribov ambiguity on local exponents



- increase n_{copy} ...

Impact of Gribov ambiguity on local exponents



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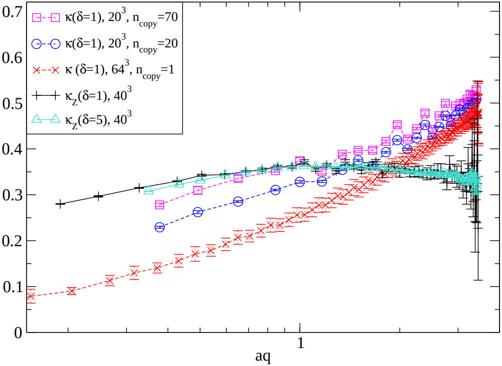

- increase n_{copy} ...

  ($\approx$ scaling can be obtained ...)

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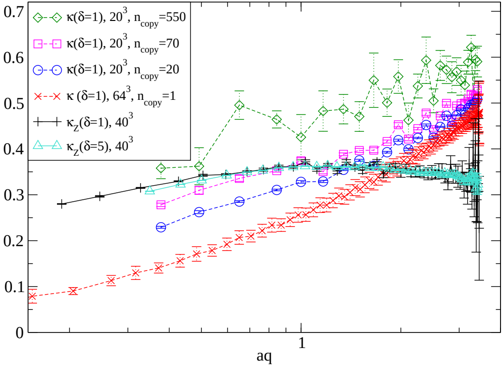

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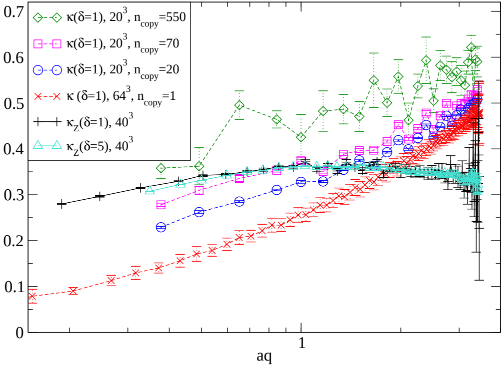

- increase n_{copy}
  $\rightsquigarrow$ 'over-scaling'
- large n_{copy}:
  κ and κ_Z don't intersect
  $\Rightarrow$ no peak of α_S^{eff}
  - remember:

$$\alpha_S^{\text{eff}} \propto \left(q^2\right)^{2(\kappa_Z - \kappa)}$$

Impact of Gribov ambiguity

Interpretation

- Gribov ambiguity more severe than expected
  - Should also hold at $\beta > 0$
- No IR solution type (scaling vs. decoupling) excluded
  - Stronger & diff. Gribov copy effect than by 'global maximization'

    (e.g. Bogolubsky et al. '05, '07, '09, Bornyakov et al. '08, '09)

    - ★ $\mathcal{O}(100\%)$ vs. 10% for ghost

      gluon: small effect vs. 20%
    - ★ 'over-scaling' vs. decoupling

      but: 'over-scaling' ruled out in continuum

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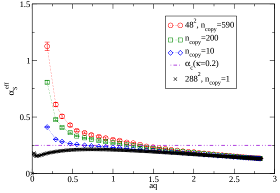

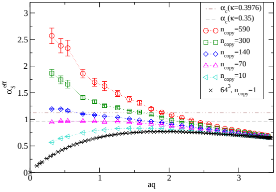

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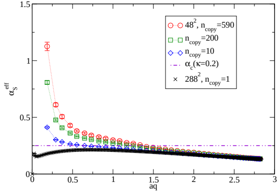

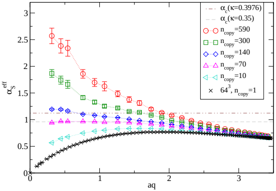

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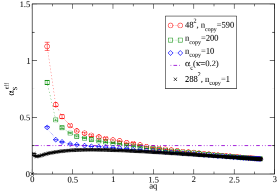
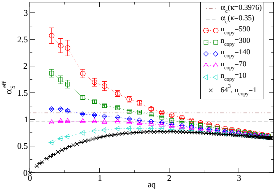
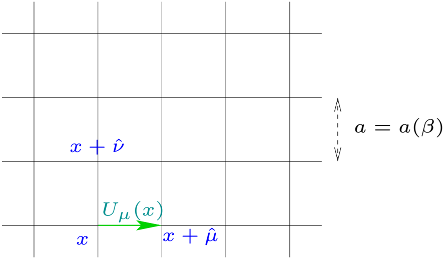




Outline

Free boundary conditions



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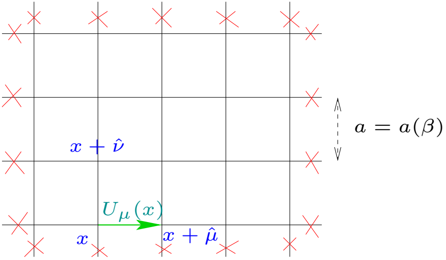

Links at boundary vanish $\rightsquigarrow$ 'finite lattice'

Free boundary conditions: Why & how?

- Free b.c. & Landau gauge $\Rightarrow$ $\boxed{D_{\mathrm{gl}}(q=0)=0}$ (Schaden/Zwanziger '94)
  - Maybe obtain scaling?!
- Caveat: Non-periodicity
  $\Rightarrow$ modify def. of D_{gl}

'Center def.'

$$D_{\mathrm{gl}}(q) \propto \sum_{a,\mu} \sum_{x} \langle A_\mu^a(x) A_\mu^a(c) \rangle \cos(q \cdot (x - c))$$

  - factor of V lacking

required

- standard g.f. (except for b.c.)
- 'first copy' approach

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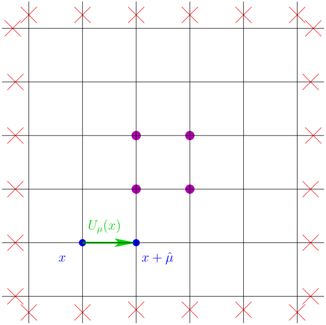
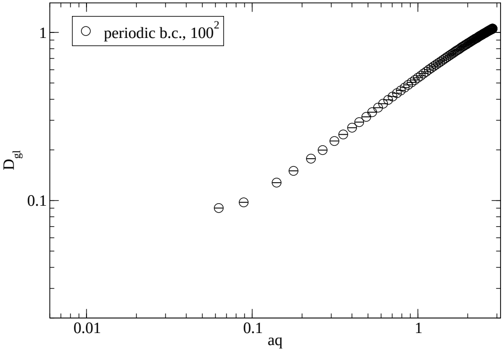


2^d 'centers'
of the lattice
instead of V sites of
the lattice

Gluon propagator from free b. c., two dimensions



$\beta = 0$ $d = 2$

- Periodic b.c.

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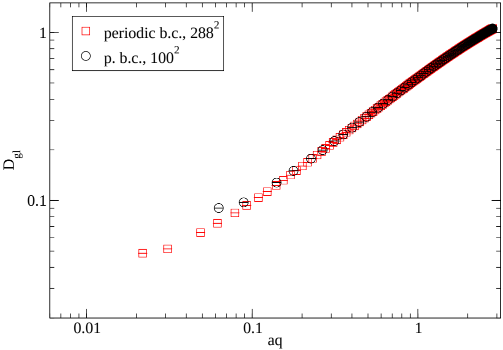

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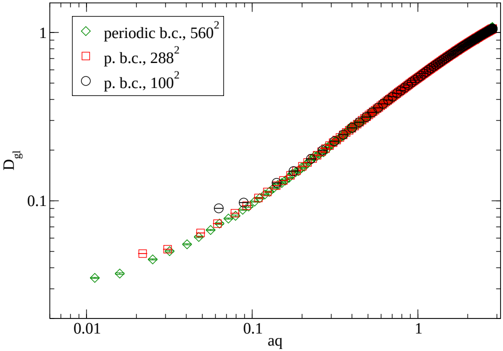

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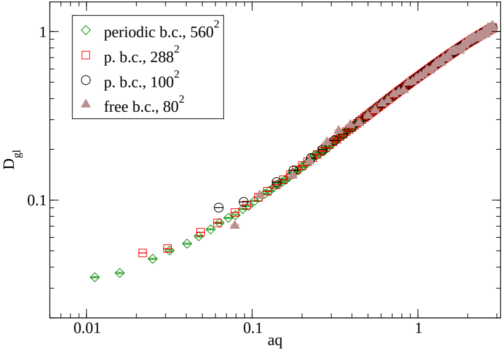

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- vs. free b.c.

(typically 500,000 config's)

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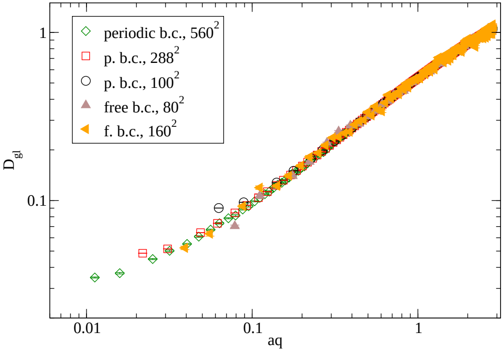

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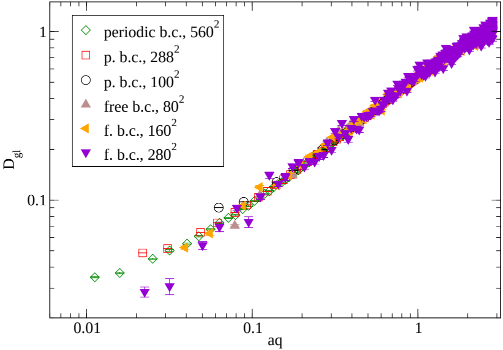


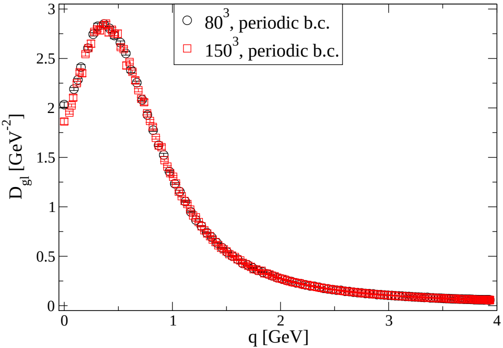
$\beta = 0$ $d = 2$

- Periodic b.c.
- vs. free b.c.

  (typically 500,000 config's)

  - consistent with same $V \to \infty$ limit

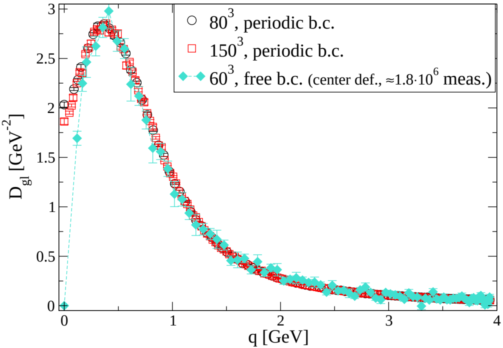
Gluon propagator from free b. c., three dimensions



finite coupling $d = 3$

Periodic b. c. ...

Gluon propagator from free b. c., three dimensions



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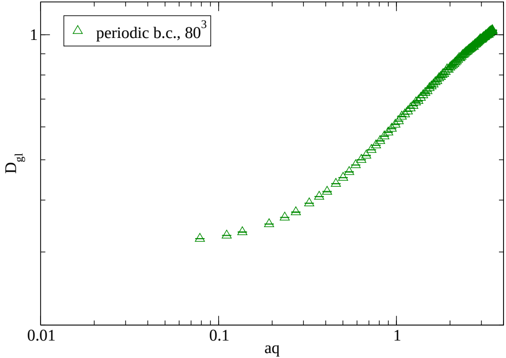
Periodic vs. free b. c.

Gluon propagator from free b. c., three dimensions



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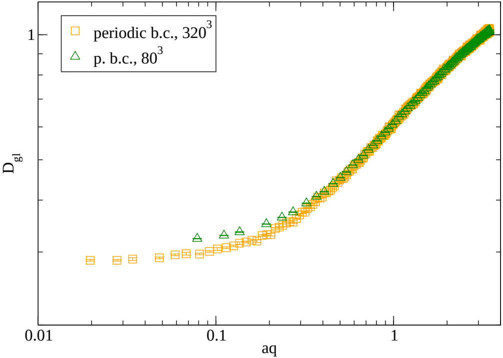
Gluon prop. with periodic b. c.

...

Gluon propagator from free b. c., three dimensions



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Gluon prop. with periodic b. c. decreases with V

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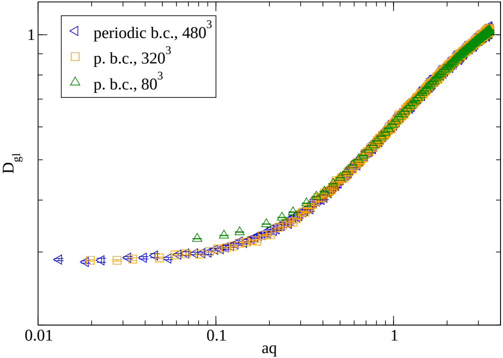

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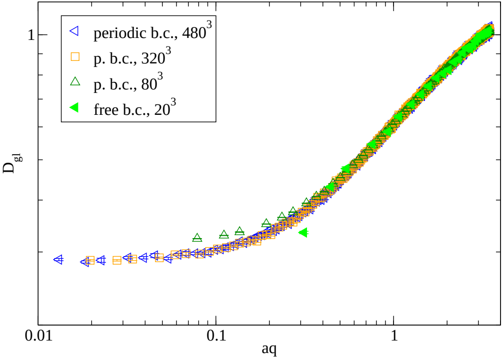

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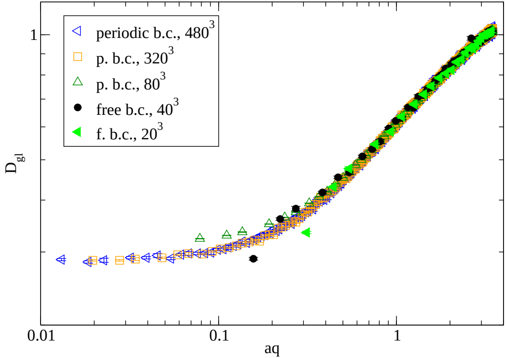

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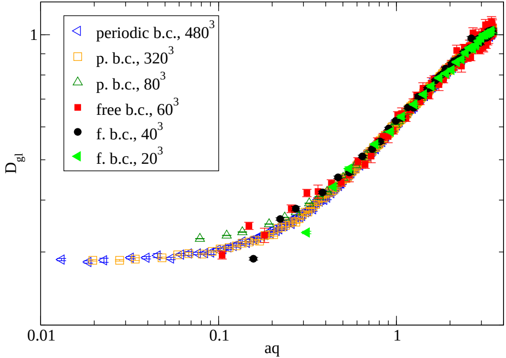


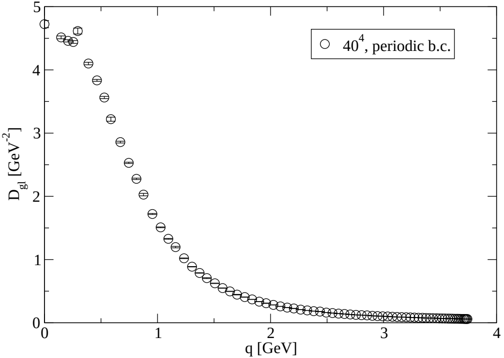
$\beta = 0$ $\quad d = 3$

$V \to \infty$: Free b. c. prop. not below periodic b. c. prop.

- $\Rightarrow$ no uniform scaling

60^3: 10^6 config's
40^3: $2.5 \cdot 10^6$ config's

Gluon propagator from free b. c., four dimensions



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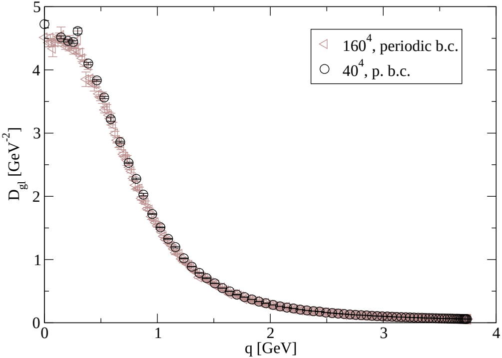

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- Periodic b.c.: rel. small volume effect

  $V = (34\ \mathrm{fm})^4$

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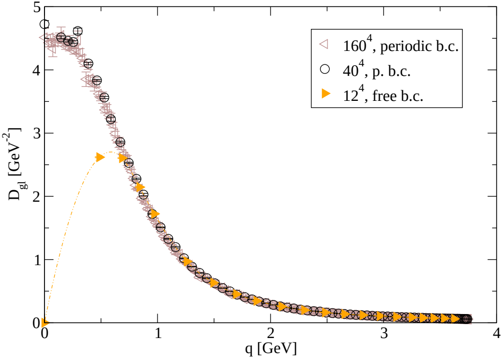

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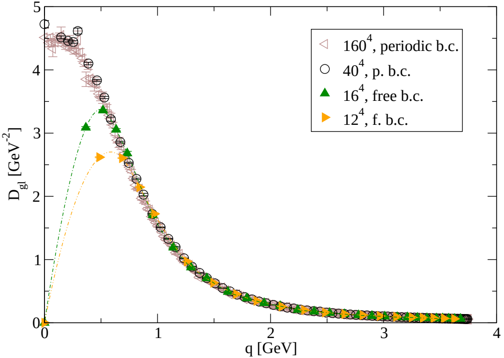

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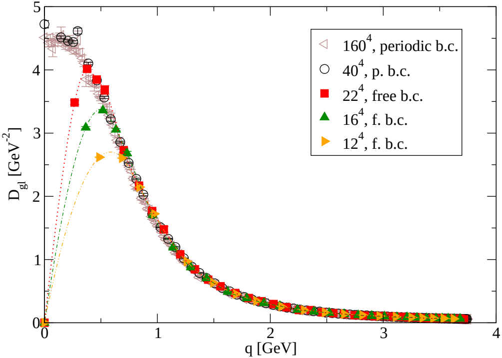

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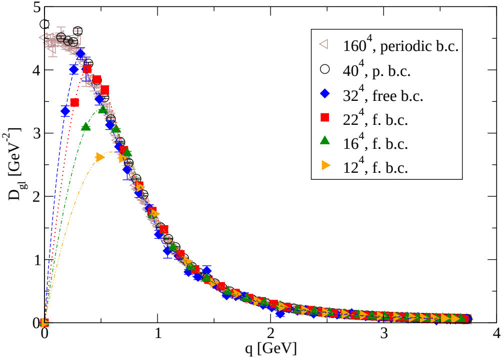

Finite coupling $d = 4$

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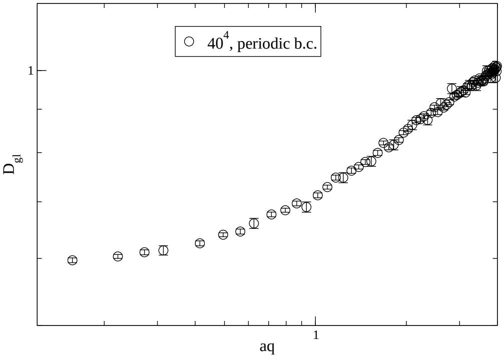
- Free b.c.: much stronger volume effect

Gluon propagator from free b. c., four dimensions



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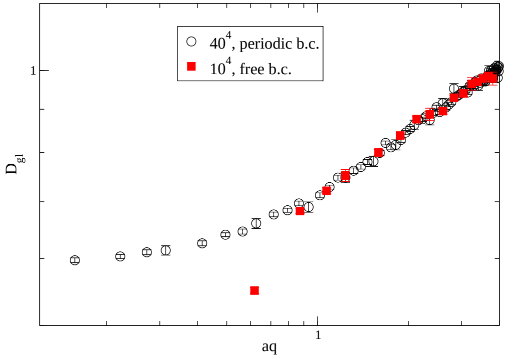

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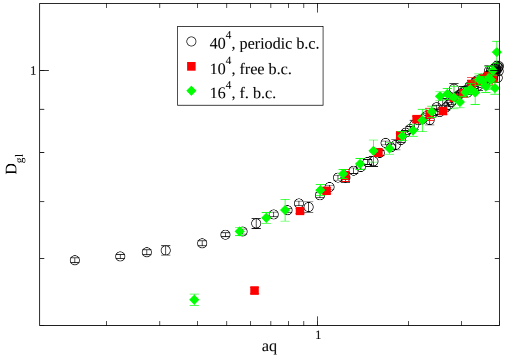

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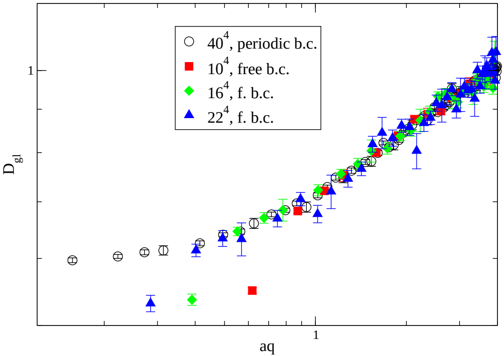

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22^4: $>$ 400, 000 config's

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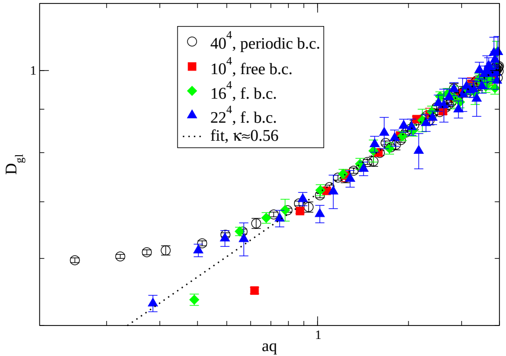

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Non-periodic gauge transformations

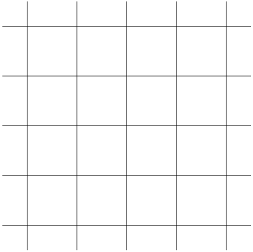
Motivation

- another way to enforce $D_{\mathrm{gl}}(0) = 0$ (for $V \to \infty$)

Implementation

- thermalize periodic config' on $(L-1)^d$ lattice
- embed in L^d lattice
- complete links
  - perpendicular links: zero
  - parallel links: periodic
- gauge fixing via non-per. g. t. on finite lattice

Non-periodic gauge transformations



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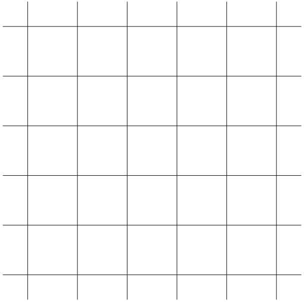

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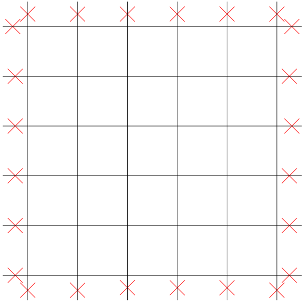

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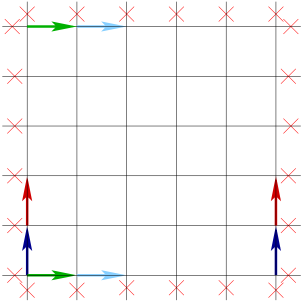

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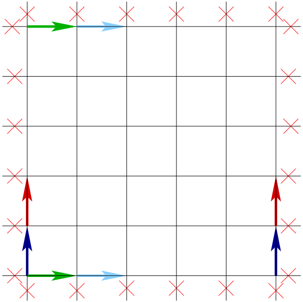
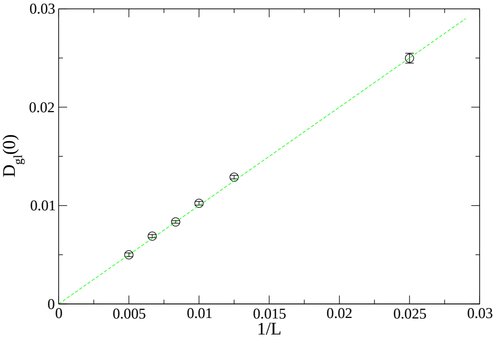


Two possibilities

- combine with ...
  - ...free b. c.
    modified D_{gl} on L^d
    - $\rightsquigarrow$ same result as usual fbc
  - ...periodic b. c.
    std. D_{gl} on $(L-1)^d$

Non-periodic gauge transformations



. . . here: combined with periodic b. c.

$\beta = 0$ $d = 3$

$$\lim_{V\to\infty} D_{\mathrm{gl}}(0) = 0$$

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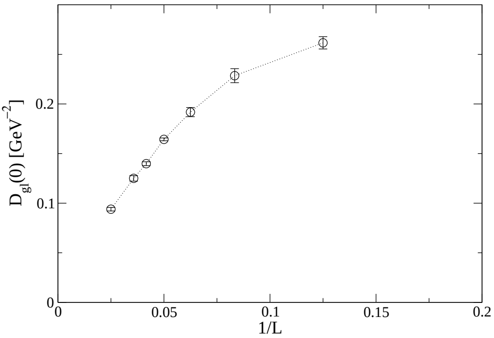

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$\beta = 2.2$ $\quad d = 4$

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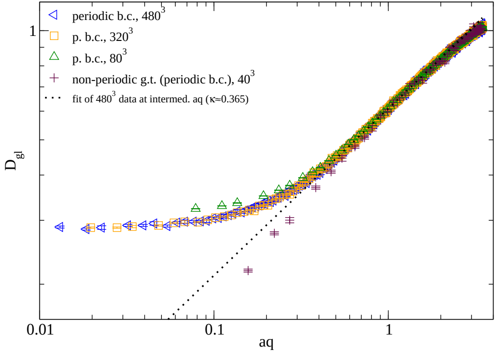
Non-periodic gauge transformations



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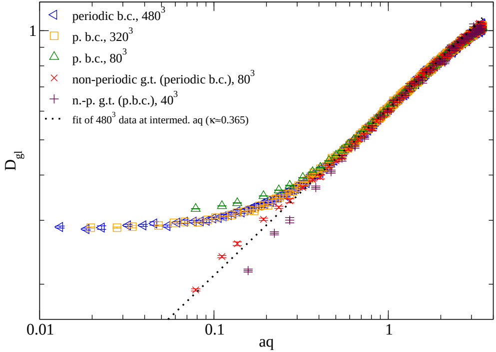
- contrasted with usual periodic b. c.

$\beta = 0$ $\qquad d = 3$

volume effect ...

Non-periodic gauge transformations



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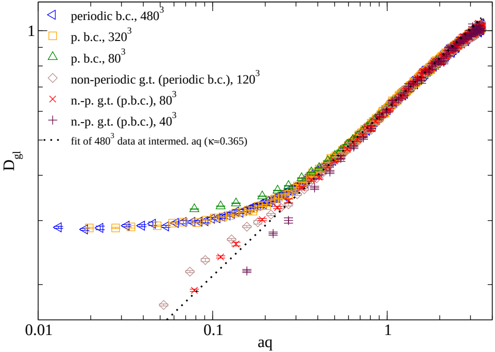

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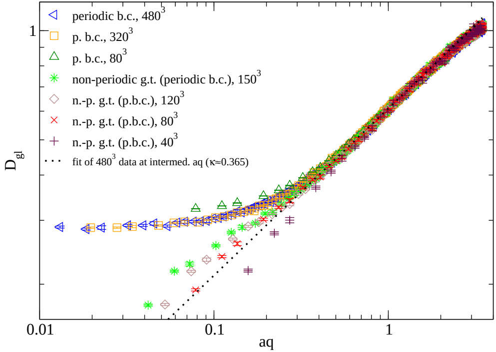

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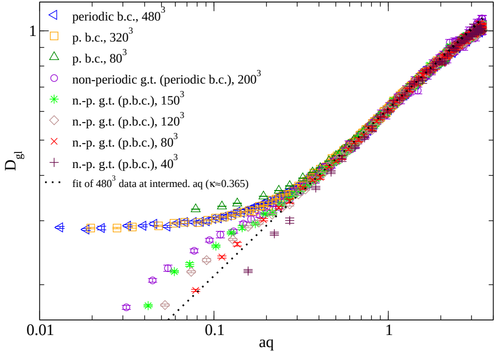

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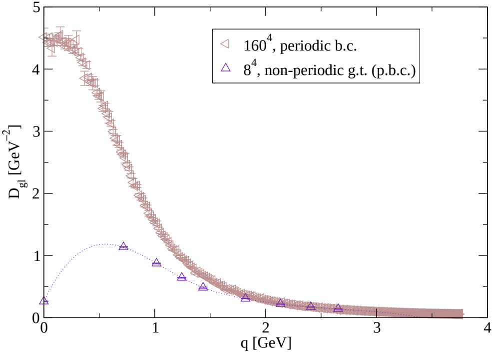
$\beta = 0$ $\quad$ $d = 3$

on large lattice:
'above' scaling solution

yet still 'below' decoupling branch

$\Rightarrow$ comparatively slow effect

Non-periodic gauge transformations



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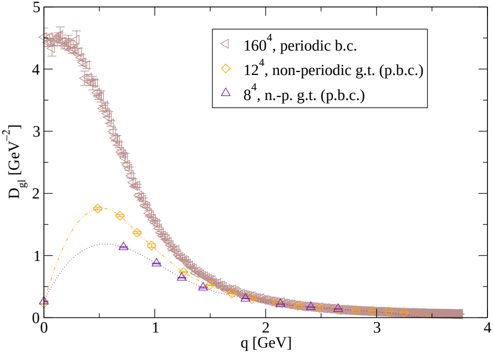
volume effect . . .

Non-periodic gauge transformations



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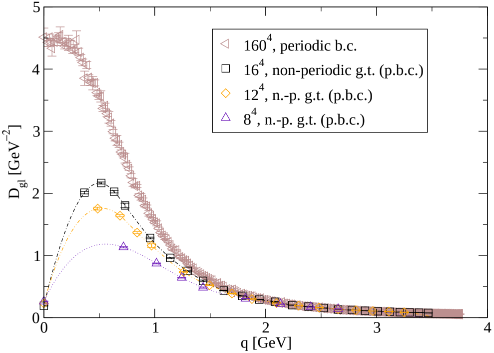

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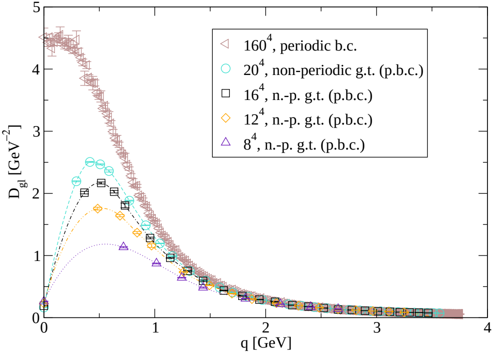

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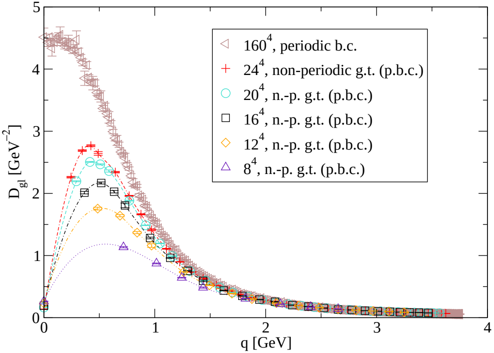

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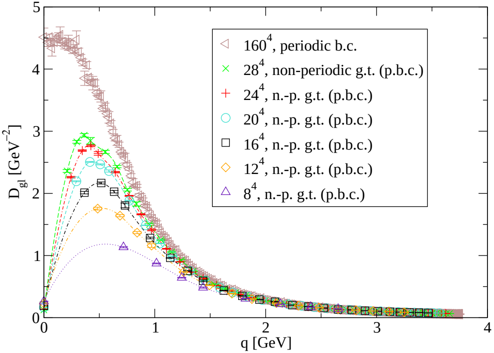

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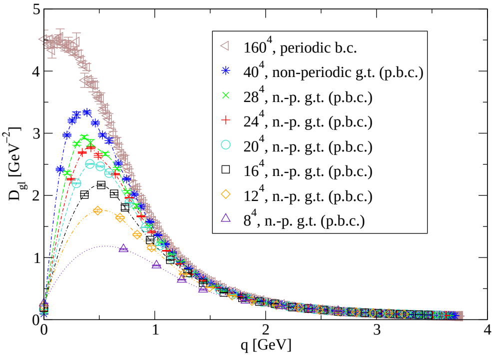

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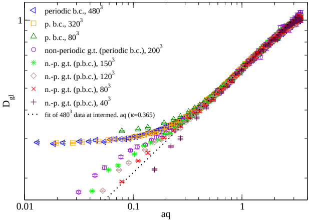
$\beta = 2.2$ $\quad d = 4$

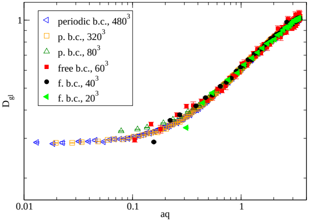
volume effect . . . slow

Non-periodic gauge transformations vs. free b. c.

non-periodic g. t. (pbc) | std. free b. c.

$\beta = 0$
$d = 3$





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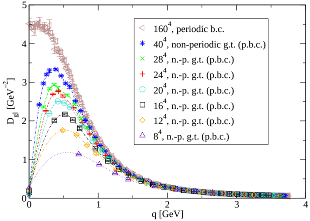

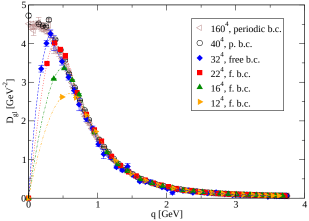

Upshot of free b. c. and non-periodic g. t.

Free b. c.

- Large V: Same result as pbc (except for smallest momenta)
- $\rightsquigarrow$ Nontrivial confirmation of standard lattice scenario

Non-periodic g. t.

- Again, no scaling behavior for $V \to \infty$
- But surprisingly slow volume effect (if combined with pbc)

Summary regarding confinement mechanism

- Stochastic quantization for gauge fixing
  - Confirmation of standard lattice scenario despite different algorithm
- Strong-coupling limit
  - Scaling branch
  - But no uniform scaling – not even in $d = 2$ (!)
- 'max-B' gauge
  - Huge effect of Gribov ambiguity
    - ★ Surprising 'over-scaling'
    - ★ No subset of solutions can be excluded yet
- Free b. c.
  - D_{gl} IR suppressed on moderate V
  - But 'decoupling branch' restored for $V \to \infty$ ($d = 3, 4$)
  - Non-periodic g. t. with periodic b. c.:
    - ★ much slower approach to dec. branch

Summary regarding confinement mechanism

Decoupling or scaling?

- Standard lattice scenario (Landau gauge; $d = 2,\ 3,\ 4$) confirmed with
  - stochastic gauge fixing
  - free boundary conditions
- Some unexpected findings
  - strong-coupling limit ($d = 2$ not so special)
  - non-periodic g. t. with periodic b. c. (finite-V effect)
- Evidence that no subset of solutions can be excluded yet (Gribov ambiguity)
  - Landau max-B gauge

Outlook regarding confinement mechanism

- Current results surprise and raise new questions
  - Landau max-B gauge
    - $V \to \infty$ and $n_{\text{copy}} \to \infty$ limit
    - $\beta > 0$ (Maas '09 ...)
  - Non-periodic g. t.
    - explanation for volume effect?
- BRST invariant lattice formulation ... (von Smekal et al. '07, '08 ...)

Outline

$T > 0$ studies

Motivation

- Recent study of $SU(2)$ & $SU(3)$ YMT in $d = 3 + 1$ (Fischer/Maas/Müller '10)
  - Information about deconfinement PT from gluon prop.
- Here: $SU(2)$ in $2 + 1$ dim.
  - finer resolution
  - PT also of 2nd order
  - information about universality class $\xi \propto \left|\frac{\beta}{\beta_c} - 1\right|^{\nu}$
    - $d = 3 + 1$: 3d Ising ($\nu \approx 0.63$)
    - $d = 2 + 1$: 2d Ising ($\nu = 1$) (Engels et al. '97)

Longitudinal resp. transverse gluon propagator

$$(D_{\mathrm{gl}})^{ab}_{\mu\nu}(q) = D^{ab}_{T}(q_0, \vec{q})P^{T}_{\mu\nu}(q) + D^{ab}_{L}(q_0, \vec{q})P^{L}_{\mu\nu}(q).$$

both transverse in full d-dim' space (Landau gauge)

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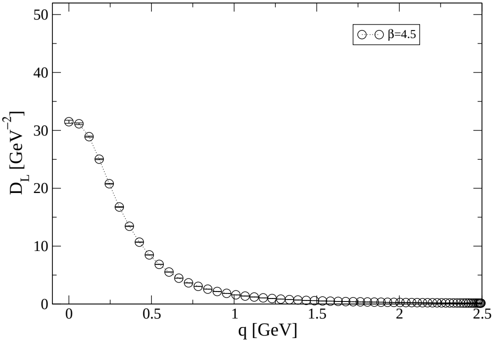
Longitudinal & transverse gluon propagator at $T > 0$



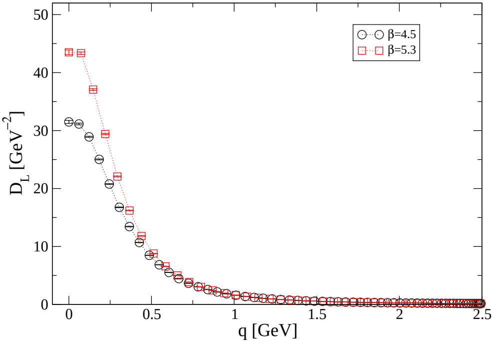
$128^2 \times 4$

$\beta_c = 6.5364$ (Edwards/von Smekal '09)

$$T = \frac{1}{L_t\, a(\beta)}$$

Longitudinal gluon propagator

Longitudinal & transverse gluon propagator at $T > 0$



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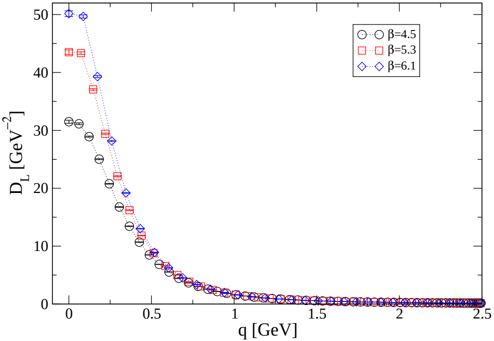

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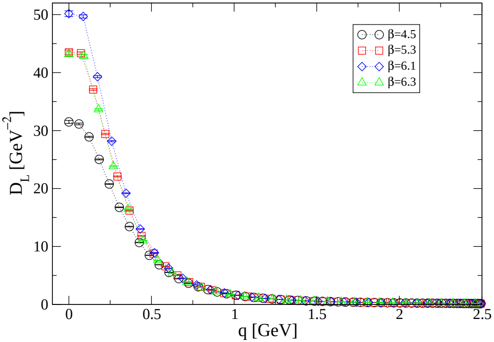

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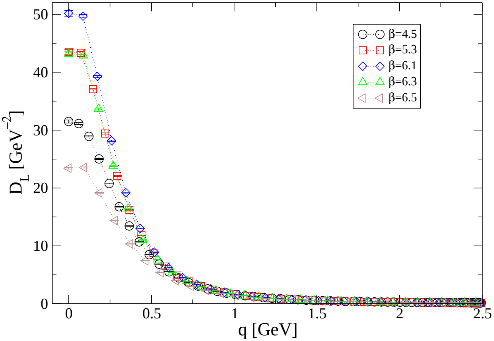

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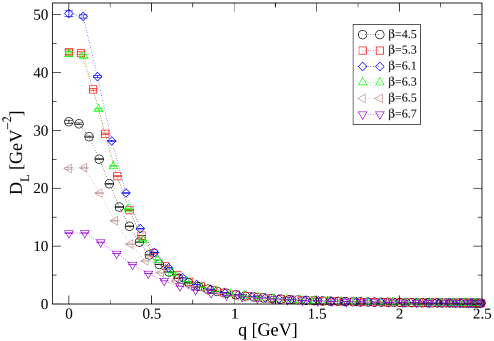

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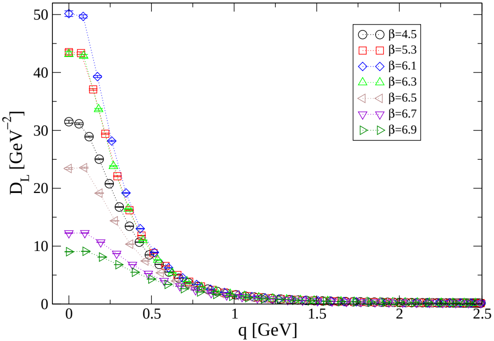

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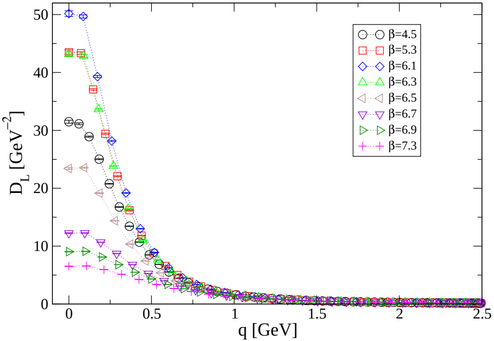

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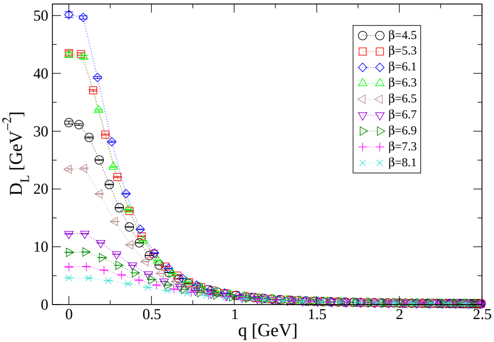

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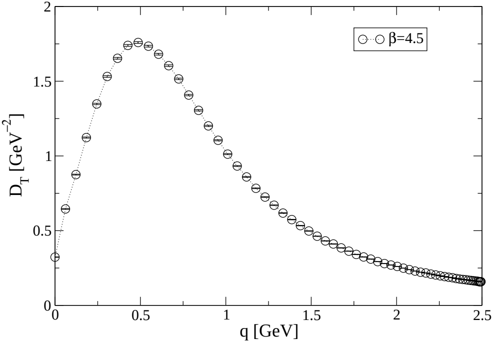
Longitudinal gluon propagator

$\Rightarrow$ sensitive to PT

Longitudinal & transverse gluon propagator at $T > 0$



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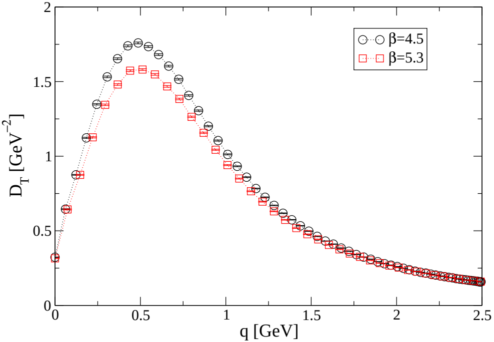
Transverse gluon propagator

Longitudinal & transverse gluon propagator at $T > 0$



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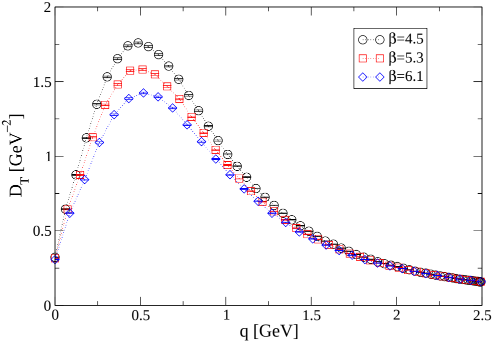

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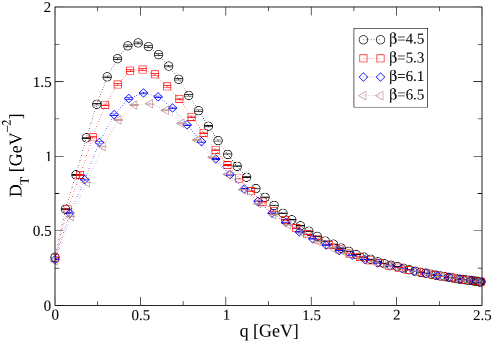

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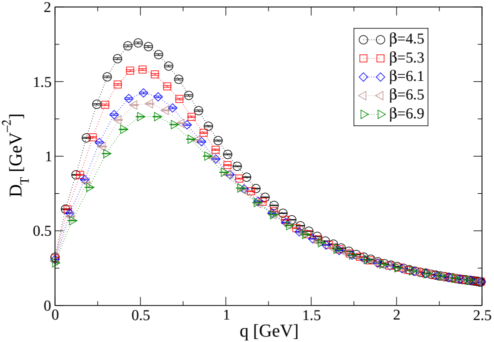

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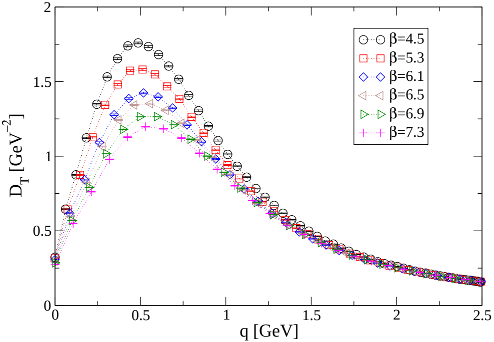

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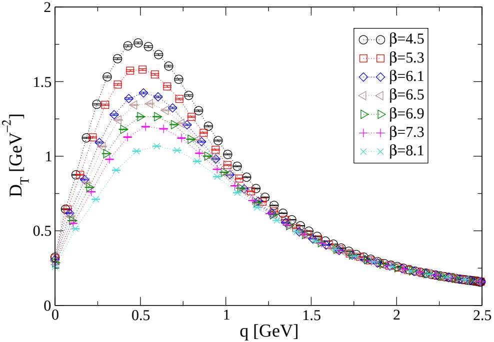

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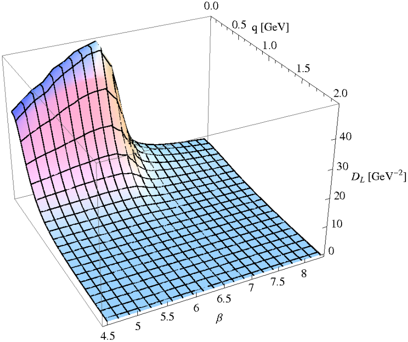
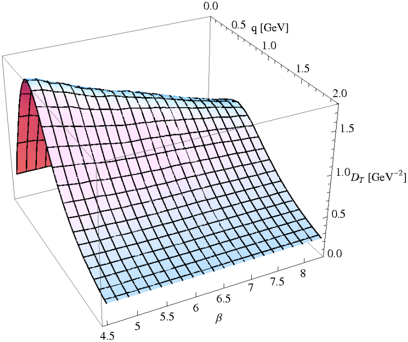
Transverse gluon propagator

$\Rightarrow$ not sensitive to PT

Longitudinal & transverse gluon propagator at $T > 0$



$D_L(q, \beta)$



$D_T(q, \beta)$

$128^2 \times 4$, $\beta_c = 6.5364$

Longitudinal & transverse gluon propagator at $T > 0$

- $\Rightarrow$ strong response to phase transition in chromoelectric sector
- more quantitative analysis:
  consider electric screening mass

$$m_L = \frac{1}{\sqrt{D_L(0)}}$$

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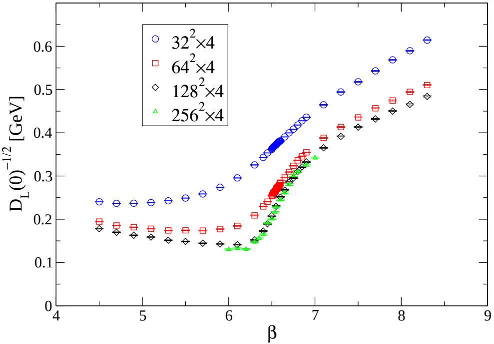
$$m_L = \frac{1}{\sqrt{D_L(0)}}$$

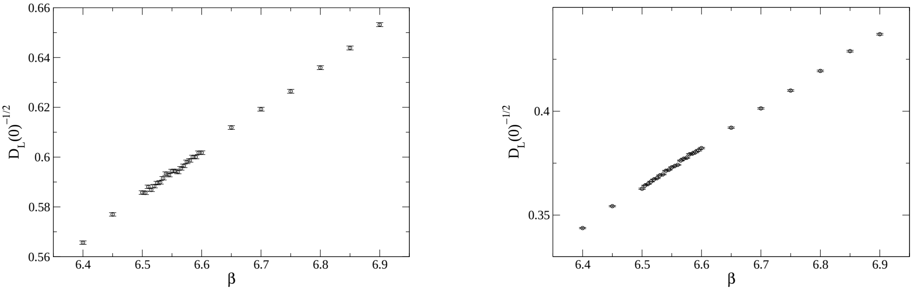
Electric screening mass vs. β



- in line with 2nd order PT
- volume effect weak beyond $L_s = 128$

now: closer look at critical behavior ...

Electric screening mass vs. β

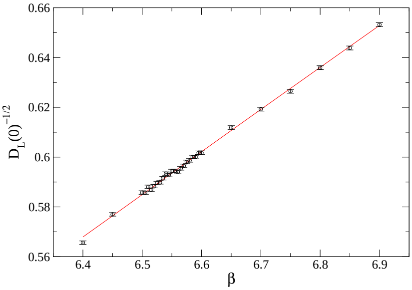


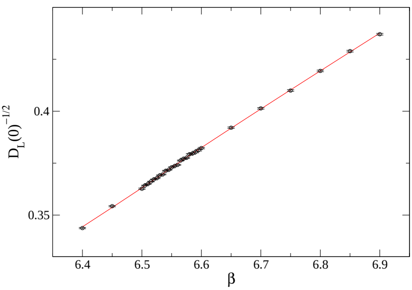
$$m_L \stackrel{!}{=} c + d\left|\frac{\beta}{\beta_c} - 1\right|^{\nu}$$

$16^2 \times 4$ $32^2 \times 4$

2d Ising: $\nu = 1$

Electric screening mass vs. β





$$m_L = c + d \left| \frac{\beta}{\beta_c} - 1 \right|^{\nu}$$

$16^2 \times 4$

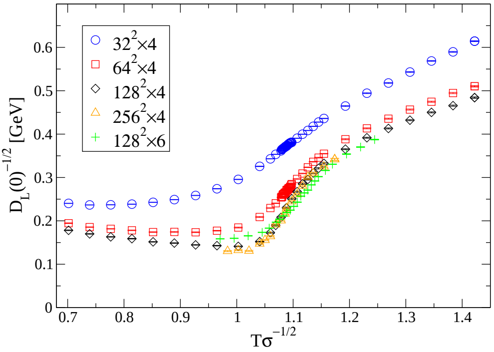
$\nu = 0.992(14)$

$32^2 \times 4$

$\nu = 0.977(6)$

2d Ising: $\nu = 1$ ✓

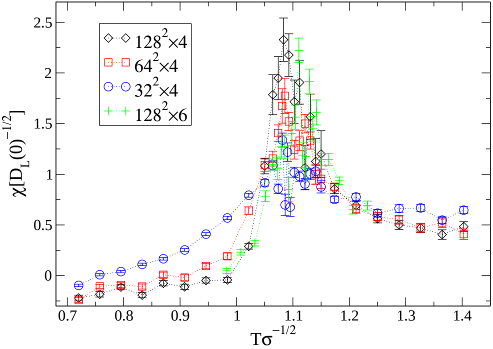
Electric screening mass vs. temperature



formerly: vs. β

set a scale via the string tension

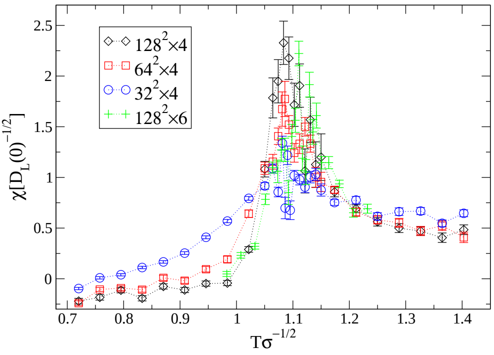
Electric screening mass vs. temperature



Susceptibility

$$\chi_T = \frac{\partial m_L}{\partial T} = \frac{\partial D_L(0)^{-1/2}}{\partial T}$$

Electric screening mass vs. temperature



Susceptibility

Results

- $T_c \approx 1.08\sqrt{\sigma}$ ($L_t = 4$)
  $T_c \approx 1.11\sqrt{\sigma}$ ($L_t = 6$)
- cp. with literature:
  $T_c \approx 1.12\sqrt{\sigma}$ ✓
  (Sternbeck/von Smekal '09; see also Liddle/Teper '08)

aim here:

- not ultimate precision,
- but evidence that the gluon propagator carries quantitative information about the critical behavior

$T > 0$ studies: upshot

- Critical behavior of $SU(2)$ YMT in $d = 2 + 1$ from the D_L
  - Correct β_c resp. T_c
  - Consistent with expected critical exponent ν
  - $\Rightarrow$ gauge-invariant information from a gauge-dependent quantity

- Caveat: different picture for larger L_t?

(Cucchieri/Mendes: recent results in $d = 3 + 1$, not yet published)

Outline

Summary regarding confinement mechanism

- Stochastic quantization for gauge fixing
  - Confirmation of standard lattice scenario despite different algorithm
- Strong-coupling limit
  - Scaling branch
  - But no uniform scaling – not even in $d = 2$ (!)
- 'max-B' gauge
  - Huge effect of Gribov ambiguity
    - ★ Surprising 'over-scaling'
    - ★ No subset of solutions can be excluded yet
- Free b. c.
  - D_{gl} IR suppressed on moderate V
  - But 'decoupling branch' restored for $V \to \infty$ ($d = 3$, 4)
  - Non-periodic g. t. with periodic b. c.:
    - ★ much slower approach to dec. branch

Summary regarding confinement mechanism

Decoupling or scaling?

- Standard lattice scenario (Landau gauge; $d = 2,\ 3,\ 4$) confirmed with
  - stochastic gauge fixing
  - free boundary conditions
- Some unexpected findings
  - strong-coupling limit ($d = 2$ not so special)
  - non-periodic g. t. with periodic b. c. (finite-V effect)
- Evidence that no subset of solutions can be excluded yet (Gribov ambiguity)
  - Landau max-B gauge

Outlook regarding confinement mechanism

- Current results surprise and raise new questions
  - Landau max-B gauge
    - $V \to \infty$ and $n_{\text{copy}} \to \infty$ limit
    - $\beta > 0$ (Maas '09 ...)
  - Non-periodic g. t.
    - explanation for volume effect?
- BRST invariant lattice formulation ... (von Smekal et al. '07, '08 ...)

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Outline

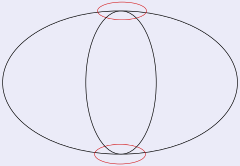
5. General backup slides

6. Sign problem and stochastic quantization: Thirring model at $\mu > 0$

Confinement scenarios

Gribov-Zwanziger scenario (Gribov '78, Zwanziger '94, '04)

- Basic idea: Configurations in vicinity of $\partial\Omega(\cap\partial\Lambda)$ account for confinement of gluons.



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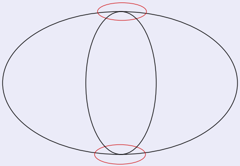

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- Implications depend on gauge...
  - In Landau gauge:
    IR vanishing of gluon propagator (horizon condition).
    ('IR slavery' scenario of *quark* confinement, $D \sim 1/q^4$, obsolete.)

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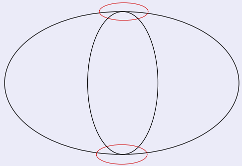

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  - In Landau gauge:
    IR vanishing of gluon propagator (horizon condition).
    Ghost dressing function divergent in the IR.

Confinement scenarios

Kugo-Ojima scenario (Kugo/Ojima '79, review: Nakanishi/Ojima '90)

- Confinement by BRST quartet mechanism
  - $\stackrel{\sim}{\approx}$ Gupta-Bleuler in QED, but applies also to *transversal* gluons
- Integral part of Kugo-Ojima criterion:
  Well-defined global color charge

  $\Longleftrightarrow$

  1. mass gap &
  2. (in Landau gauge) IR enhanced ghost propagator

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Confinement scenarios and correlation functions

Implications in a nutshell

- Kugo–Ojima
  - (BRST quartet mechanism, $\approx$ Gupta-Bleuler in QED)

  $\kappa > 0$
- Gribov–Zwanziger (stronger implications)
  - (confinement by config's close to 1st Gribov horizon)

  $\boxed{\kappa > 0}$ for ghost, $\kappa > 1/2$ for gluon (i.e. $\boxed{\kappa_A < -1}$)

  I.e.:
  - Ghost dressing function IR divergent
  - Gluon propagator IR vanishing

Basics of stochastic quantization

Langevin dynamics

- Stochastic quantization (Parisi/Wu '81): d-dim. quantum system $\hat{=}$ $(d+1)$-dim. classical system with random fluctuations
- Stochastic process in additional time
  Equilibrium distrib. = path-integral measure, i. e. $\exp(-S)$ (Euclidean space)
- Represented by Langevin eq.

$$dx = \underbrace{K_x d\theta}_{\text{drift}} + \underbrace{\eta(\theta) d\theta}_{\text{diffusion}}$$

with Gaussian white noise η $\qquad \langle \eta(\theta)\eta(\theta')\rangle = 2\delta(\theta - \theta')$

  - related to Wiener process, describing Brownian motion

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- Correct convergence?
  - $S \in \mathbb{R}$: ✓ (if S has lower bound)
  - $S \in \mathbb{C}$: caution required
    (runaway trajectories or wrong convergence possible),
    but maybe useful where other methods fail!

Lattice implementation of stochastic quantization

Dynamics

Lattice Langevin eq. with step size ε (e. g. Batrouni et al. '85)

$$U_\mu(x) \to \exp(i\sigma^a F^a_{\mu x}) U_\mu(x)$$

$$\text{with force } F^a_{\mu x} = \underbrace{i\varepsilon \nabla_{x\mu a} S[U]}_{\text{drift}} + \underbrace{\sqrt{\varepsilon}\eta_{x\mu a}}_{\text{diffusion}}$$

Gauge fixing

...via Zwanziger's drift term (Rossi et al. '88)

$$\boxed{\Omega(x) = \exp\left(\ldots \cdot \sigma^a \frac{\Delta^a}{\alpha}\right)}$$

$$\left(\text{general lattice g.t.: } U_\mu(x) \to \Omega(x) U_\mu(x) \Omega^\dagger(x+\hat{\mu})\right)$$

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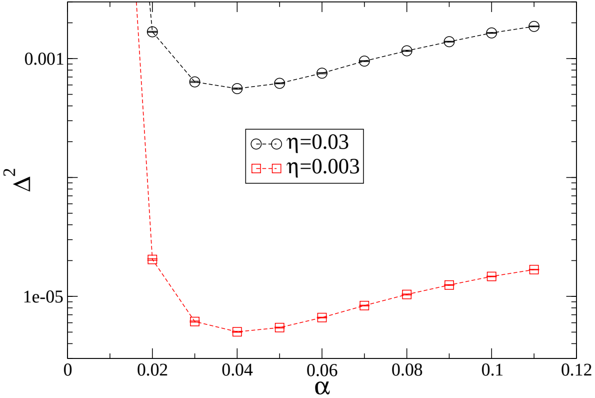
$$\boxed{\Omega(x) = \exp\left(\ldots \cdot \sigma^a \frac{\Delta^a}{\alpha}\right)}$$

$\Delta^a \hat{=} \partial_\mu A^a_\mu, \quad \alpha$: gauge parameter

Lattice implementation of stochastic quantization

- Alternative lattice formulation of the dynamics:
  Random walk:
  $$U'_\mu(x) = \exp(iA^b_\mu \sigma^b) U_\mu(x)$$
  with contributions $\pm\eta$ (fixed size) to $A^b_\mu(x)$,
  accepted with probability
  $$p = \frac{1}{2}\left(1 \pm \tanh\left(\frac{1}{2}\eta i \nabla_{x\mu a} S[U]\right)\right)$$

Gauge fixing parameter and accuracy of g. f.



- $\exists$ optimal α for given parameters (small, but non-zero)

here: random walk

Propagators from stochastic gauge fixing

Bottom line

Confirmation of

standard lattice results . . .

2D: scaling, 3&4D: decoupling

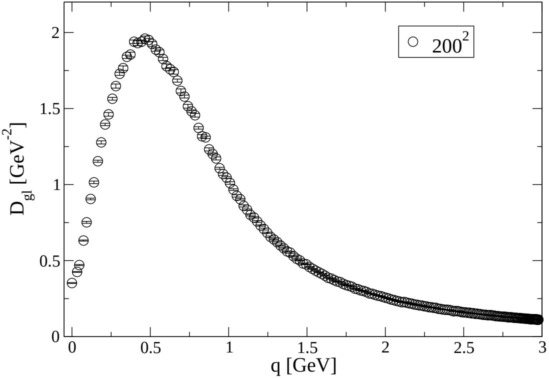
e.g.

- 2D: Maas '07
- 3D: Cucchieri/Mendes '03
- 4D: Sternbeck et al. '05, '06, Bogolubsky et al. '07, '08, '09, Cucchieri/Mendes '07 . . .

. . . with alternative

gauge fixing method

Propagators from stochastic gauge fixing



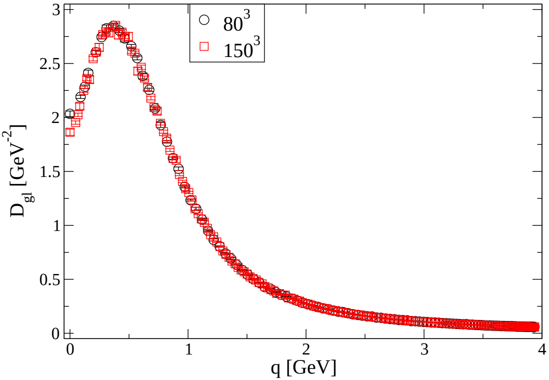
Gluon propagator

2D: compatible with IR scaling

- $\kappa \approx 0.2$, as expected

But: $\beta = 0$ results raise new questions

Propagators from stochastic gauge fixing

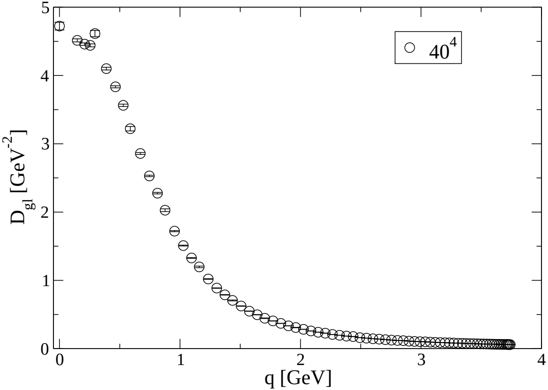


Gluon propagator

3D: Peak, but IR non-zero
$\rightsquigarrow$ decoupling

But: at $\beta = 0$, 3D similar to 2D

Propagators from stochastic gauge fixing

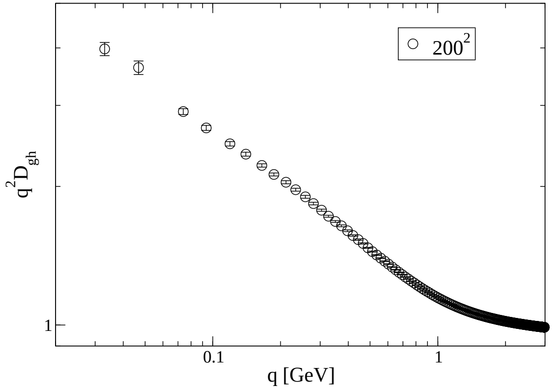


Gluon propagator

4D: IR non-zero

$\rightsquigarrow$ decoupling

Propagators from stochastic gauge fixing



Ghost dressing function

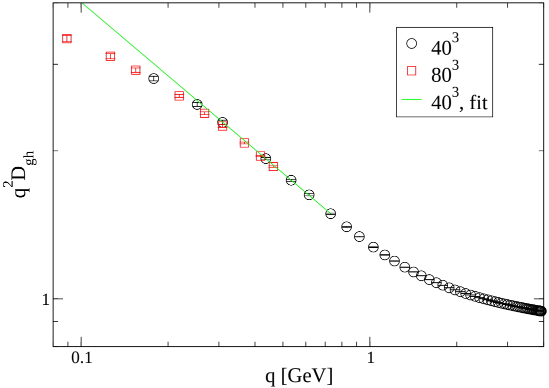
2D: IR divergent

$\kappa \approx 0.2$

- as expected for scaling
- consistent with gluon data

But: different picture at $\beta = 0$

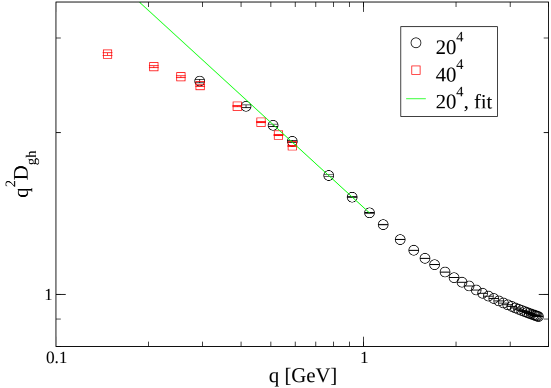
Propagators from stochastic gauge fixing



Ghost dressing function

3D: tends towards decoupling

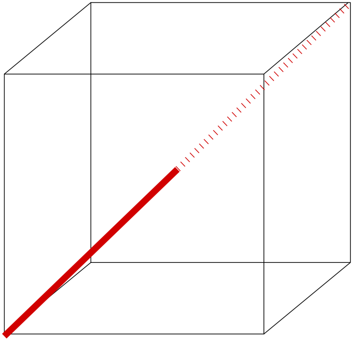
Propagators from stochastic gauge fixing



Ghost dressing function

4D: tends towards decoupling

Discretization artifacts at $\beta = 0$



Idea

- Usually: cylinder cut $\rightsquigarrow$ momenta near diagonal
- Now cp. with on-axis momenta
- $\Rightarrow$ Assess effect of breaking rotational invariance

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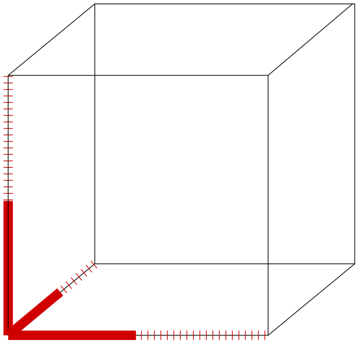

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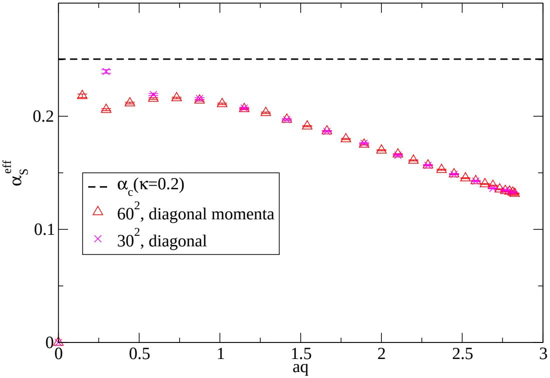

$d = 2$

- Diagonal mom.

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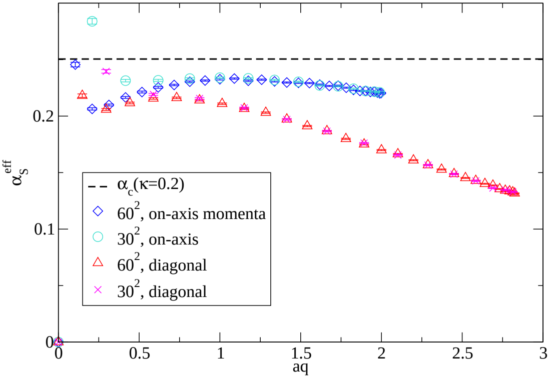
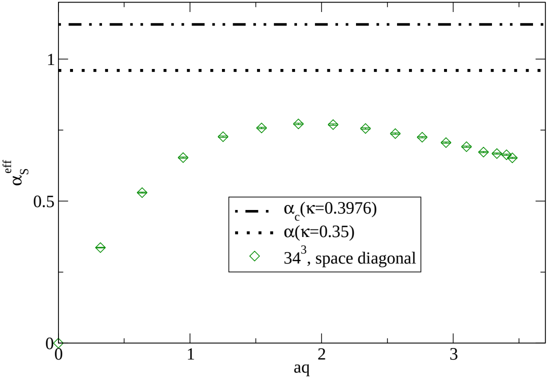


$d = 2$

- Diagonal mom.
- On-axis mom.
  - closer to scaling behav.

Discretization artifacts at $\beta = 0$



$d = 3$

- Space-diag. mom.

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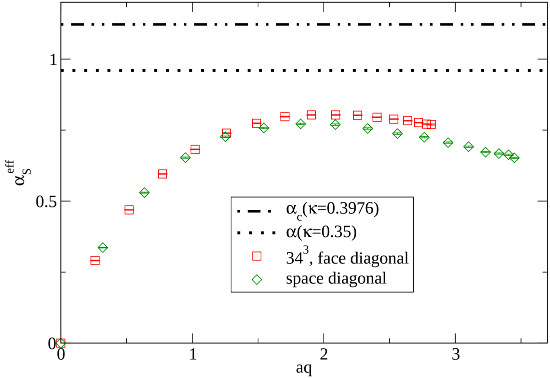

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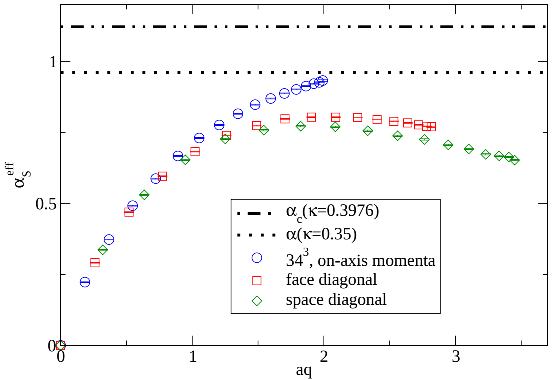

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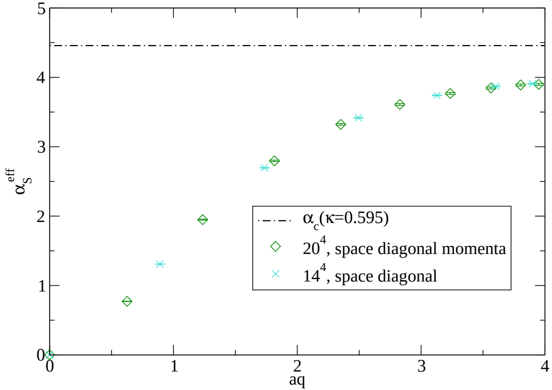

$d = 4$

- Space-diag. mom.

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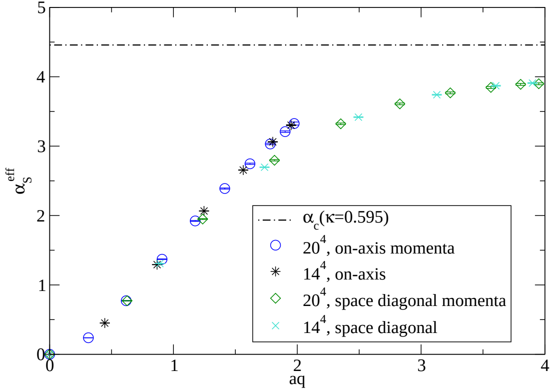
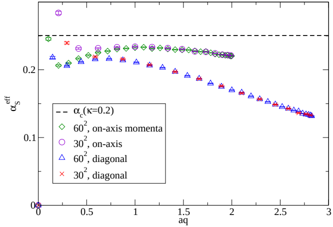
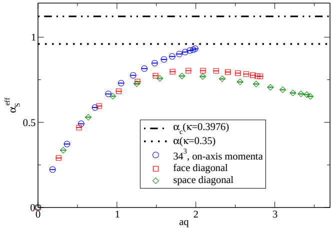


$d = 4$

- Space-diag. mom.
- On-axis mom.
  - Effect weaker at larger d

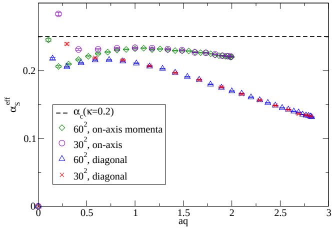
Discretization artifacts at $\beta = 0$

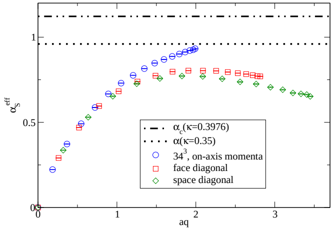




- $\Rightarrow$ Discretization problem at large aq
- Now for a stronger effect: Gribov copies . . .

Discretization artifacts at $\beta = 0$





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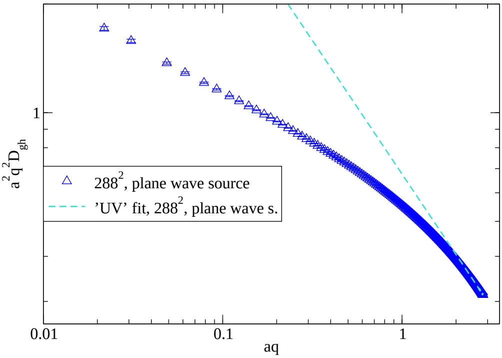
Strong-coupling limit: Two dimensions



Ghost propagator

- plane-wave source $\rightsquigarrow$ precise result ($\kappa \approx 0.37$)
- local κ
  - monotonically rising
  - in general $\kappa \neq \kappa_Z$ $\Rightarrow$ no scaling
- cp. with point source

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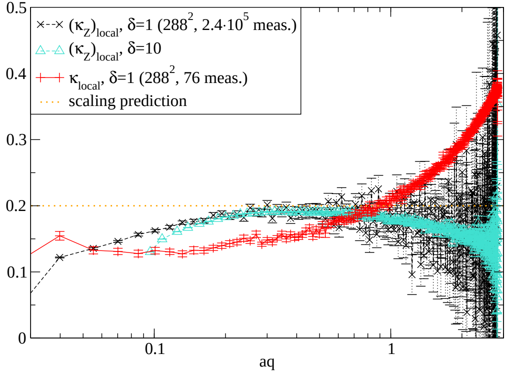

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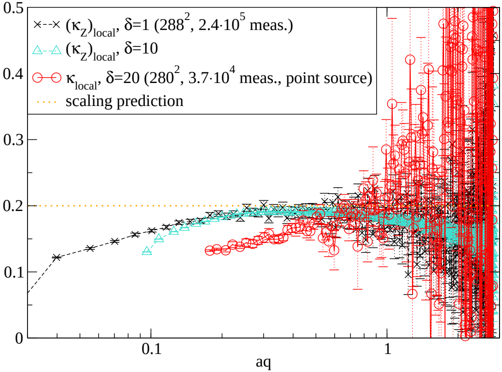

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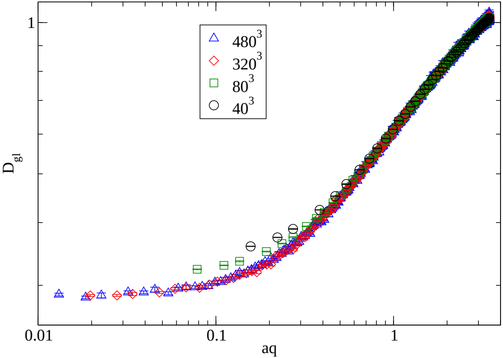

Gluon propagator

- . . . at different V
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  - qualitatively similar to 2D case

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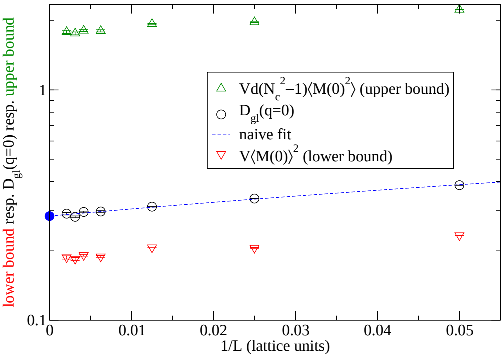

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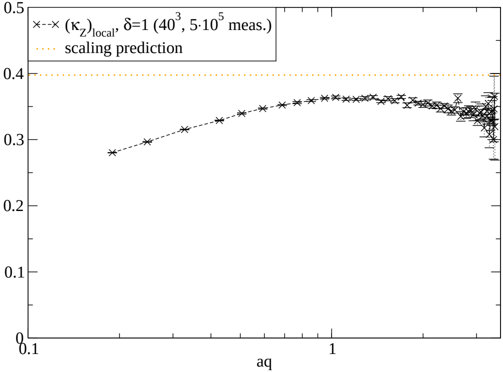

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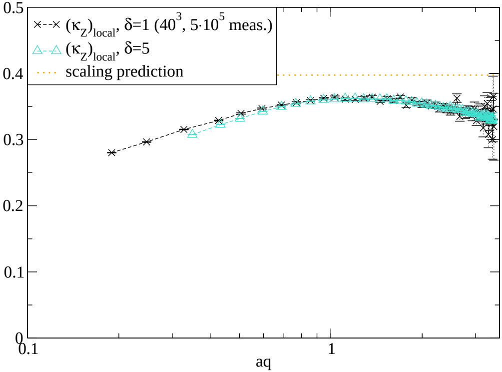

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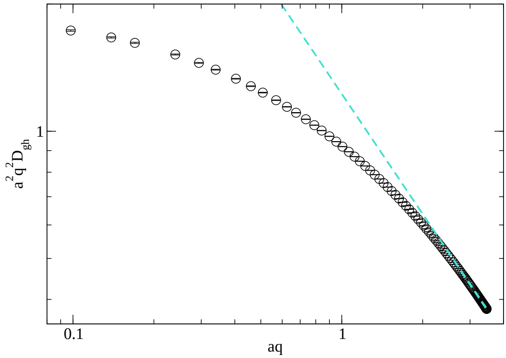

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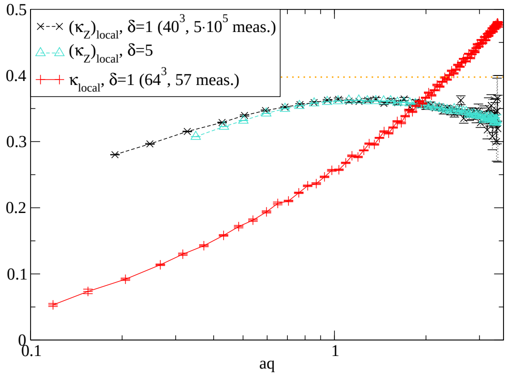

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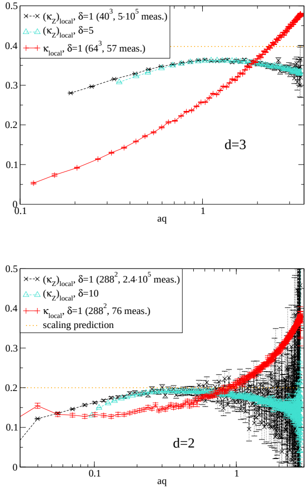
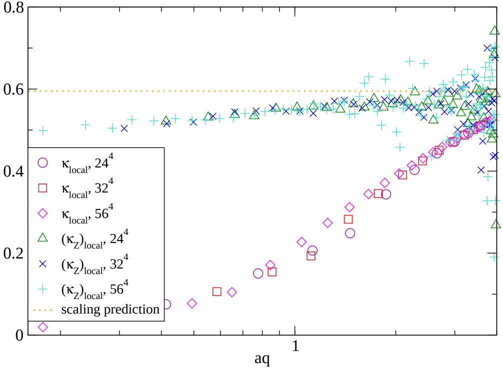


Ghost propagator

- ghost dressing function
- local κ
  - monotonically rising
  - $\kappa \neq \kappa_Z$
  - resembles 2D case

Strong-coupling limit: Four dimensions



local κ & κ_Z

- additional analysis of previous 4D data

  (Sternbeck/von Smekal '08)

- relation to effective running coupling ...

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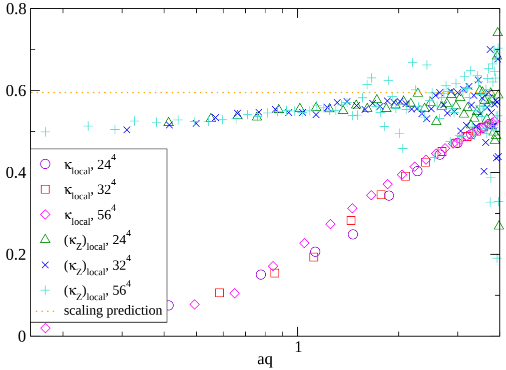

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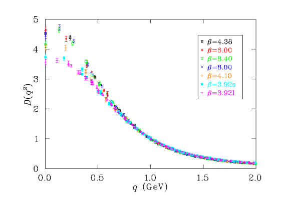
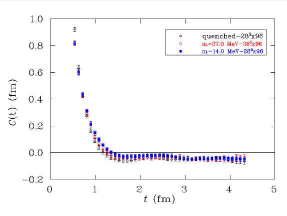
Importance of IR asymptotics

What κ may tell about confinement

- IR asymptotics of $D_{\mathrm{gl}}(q^2)$, $D_{\mathrm{gh}}(q^2)$
  $\rightsquigarrow$ Test predictions of confinement scenarios
- E.g.: Violation of reflection positivity by IR non-divergent gluon propagator (lattice: Bowman et al. '07 (4D), Cucchieri et al. '05 (3D))

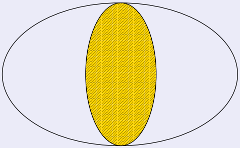




How to sample the configuration space

Possible requirements on the algorithm

Find either of the following:



- Fundamental modular region Λ (free of Gribov copies)
  $\hat{=}$ NP-hard optimization problem
- Entire Gribov region Ω with Faddeev–Popov weight
  - (Conjecture: Ω & Λ equiv. for expectation values (Zwanziger '04))
  - Restriction to Ω automatically (by g.f.)
    precise distribution nontrivial
- Bias towards Gribov horizon $\partial\Omega$ (Gribov–Zwanziger sc.: config's near $\partial\Omega \cap \partial\Lambda$ account for confinement)

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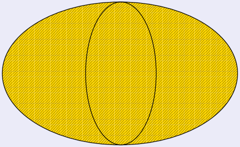

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    - ★ Faddeev–Popov operator is Hessian of $\int d^4x |{}^{\omega}A|^2$, which is minimized by numerical Landau g. f.;

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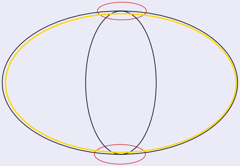

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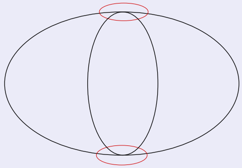
Confinement scenarios

Gribov–Zwanziger scenario (Gribov '78, Zwanziger '94, '04)

- Basic idea: Configurations in vicinity of $\partial\Omega(\cap\partial\Lambda)$ account for confinement of gluons.



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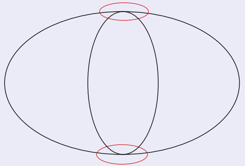

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    ('IR slavery' scenario of *quark* confinement, $D \sim 1/q^4$, obsolete.)

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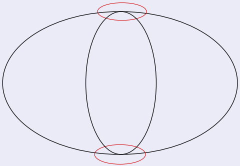

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    Ghost dressing function divergent in the IR.

Confinement scenarios

Kugo–Ojima scenario (Kugo/Ojima '79, review: Nakanishi/Ojima '90)

- Confinement by BRST quartet mechanism
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- Integral part of Kugo–Ojima criterion:
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Free boundary conditions

Sketch of proof of $D_{\mathrm{gl}}(0) = 0$

in general:

$$D_{\mathrm{gl}}(0) \propto \sum_{x,y} \langle A_\mu^a(x) A_\mu^a(y) \rangle \ \text{resp.} \ \sum_x \langle A_\mu^a(x) A_\mu^a(c) \rangle$$

$$Q_\nu(x_\nu) := \sum_{\substack{x \\ x_\nu \text{ fixed}}} A_\nu(x)$$

$$\partial_\mu A_\mu(x) = 0 \quad \Rightarrow \quad \sum_{\substack{x \\ x_\nu \text{ fixed}}} \partial_\mu A_\mu(x) = 0 \quad \Rightarrow \quad Q_\nu(x_\nu) - Q_\nu(x_\nu - 1) = 0$$

for free b. c.: $Q_\nu(0) = Q_\nu(L-1) = 0 \quad \Rightarrow \quad Q_\nu(x_\nu) = 0$

$$\Rightarrow D_{\mathrm{gl}}(0) = 0 \ \square$$

Outline

Thirring model: motivation

warming up for full QCD ...

- fermionic model with a sign problem
- initially: investigate "Silver Blaze" problem
  - below μ_c, observables independent of μ
  - onset differs from full to phase quenched theory ($S \to \mathrm{Re}\, S$)

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Sign problem in a nutshell

Sign problem

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$\det M$, thus S, complex

standard MC (importance sampling) not feasible

Possible solutions

- Reweighting (Barbour et al. '98; Fodor/Katz '02)
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Lattice model

$$S = \sum_{x,y} \sum_{i=1}^{N} \bar{\chi}_i(x) M_{x,y} \chi_i(y) + \frac{N}{4g^2} \sum_{x,\nu} A_\nu^2(x)$$

fermion matrix

$$M_{x,y} = m\delta_{x,y} + \frac{1}{2} \sum_{\nu} \eta_\nu(x) \Big[(1 + iA_\nu(x))\, e^{\mu\delta_{\nu,0}} \delta_{y,x+\hat{\nu}} - \\ - (1 - iA_\nu(y))\, e^{-\mu\delta_{\nu,0}} \delta_{y,x-\hat{\nu}} \Big]$$

(apbc in temporal direction)

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Complexify the auxiliary field,

$$A_\nu(x) \longrightarrow A_\nu^{\mathrm{R}}(x) + iA_\nu^{\mathrm{I}}(x).$$

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$$A_\nu^{\mathrm{R}}(x, n+1) = A_\nu^{\mathrm{R}}(x, n) - \varepsilon \,\mathrm{Re}\, \frac{\delta S_{\text{eff}}}{\delta A_\nu(x, n)} + \sqrt{\varepsilon}\, \eta_\nu(x, n)$$

$$A_\nu^{\mathrm{I}}(x, n+1) = A_\nu^{\mathrm{I}}(x, n) - \varepsilon \,\mathrm{Im}\, \frac{\delta S_{\text{eff}}}{\delta A_\nu(x, n)}$$

$(\theta = n \cdot \varepsilon)$

Observables

- chiral condensate

$$\langle\bar{\chi}\chi\rangle = \frac{1}{V}\left\langle \operatorname{tr} M^{-1}\right\rangle$$

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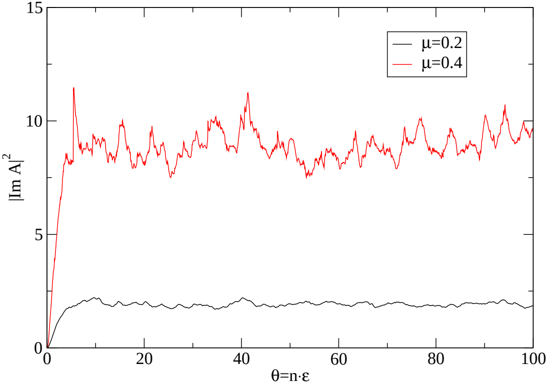
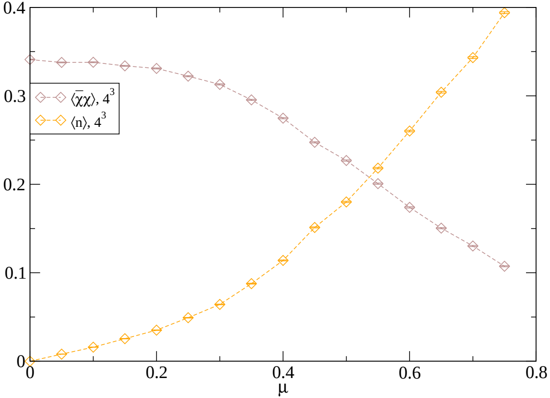
No runaways observed



Im A remains bounded during Langevin evolution

'Silver Blaze' problem



Expectation values at $\mu > 0$.
Increase lattice size . . .

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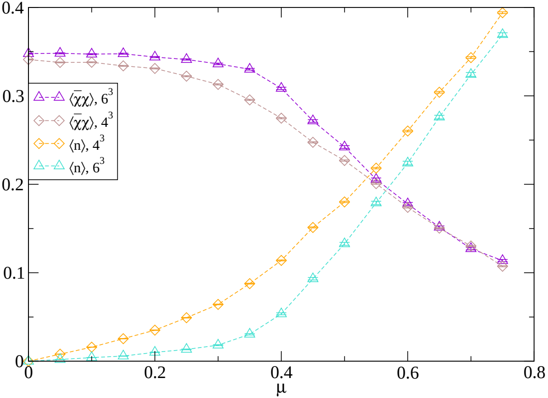

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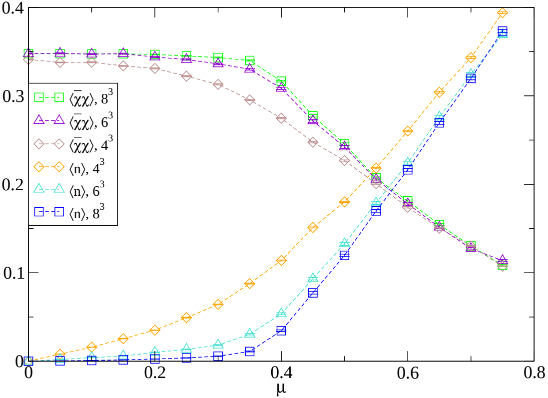

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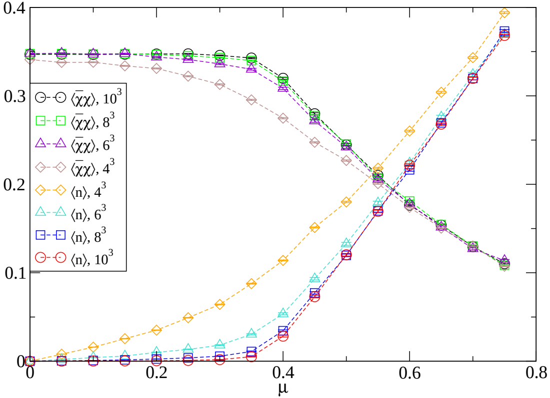

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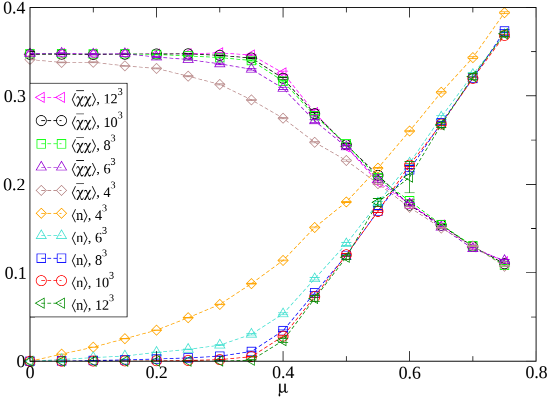

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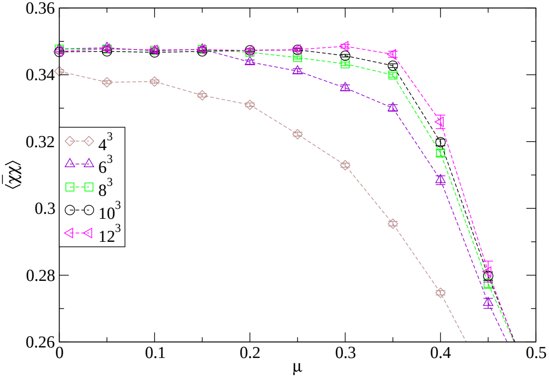

Evidence for 'Silver Blaze' behavior in thermodyn. limit ✓

But: still need cp. with phase quenched theory in order to rule out 'fake onset'

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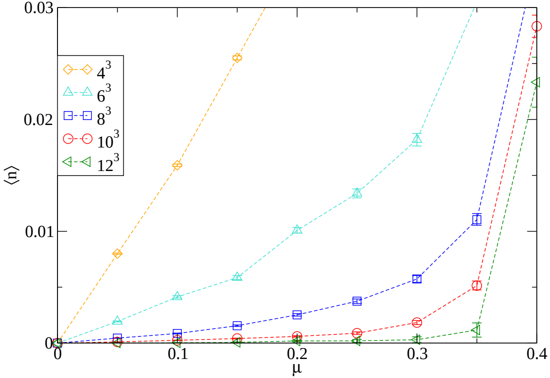

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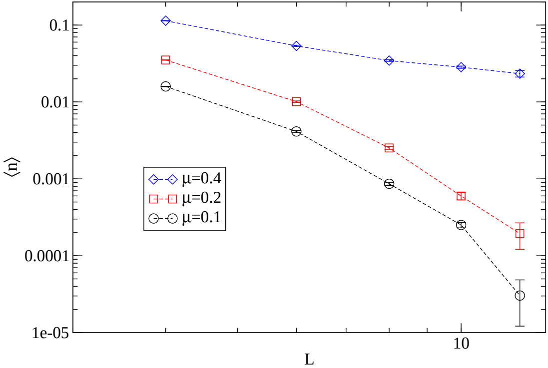

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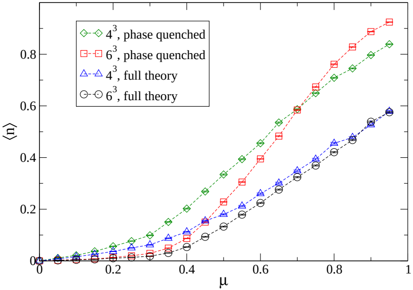

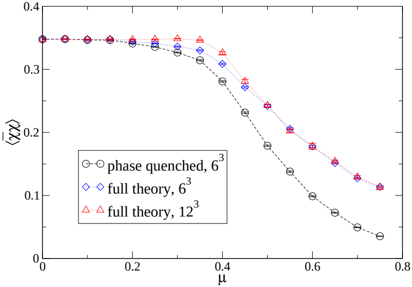

clear differences between full & ph'q' theory at $\mu \neq 0$

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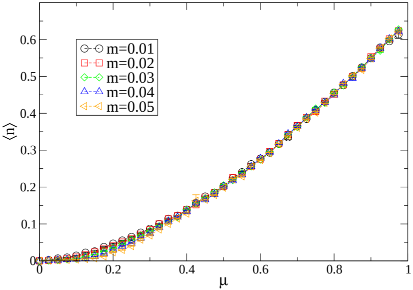
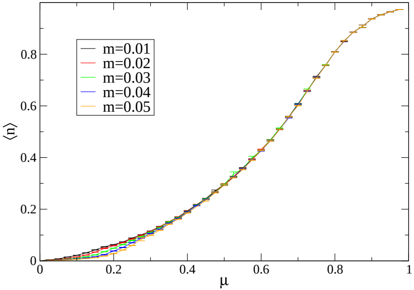
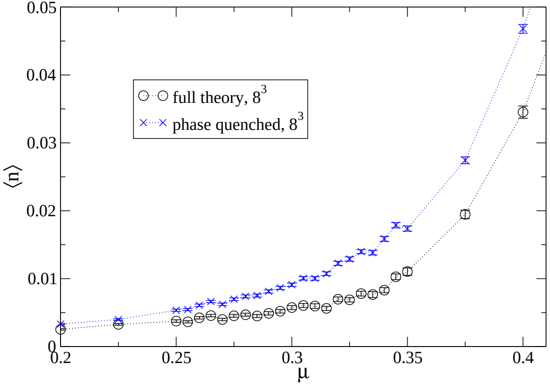




clear differences between full & ph'q' theory at $\mu \neq 0$

Cp. with phase quenched theory (II): near the onset



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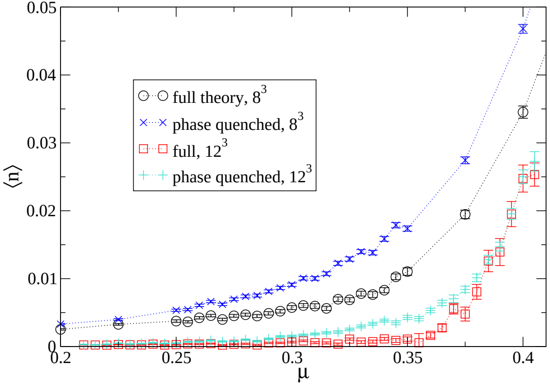

no evidence that full & ph'q' μ_c differ in thermodyn. limit

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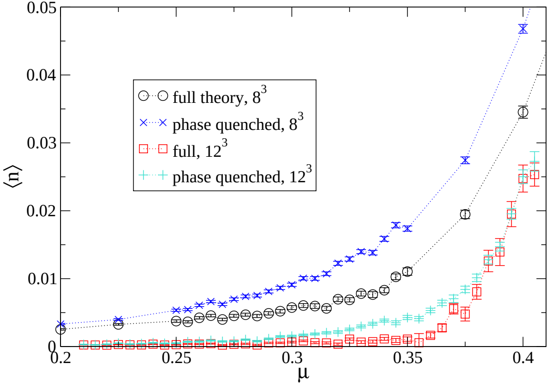

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here, $m = 0.2$ ('large') consider smaller masses ...

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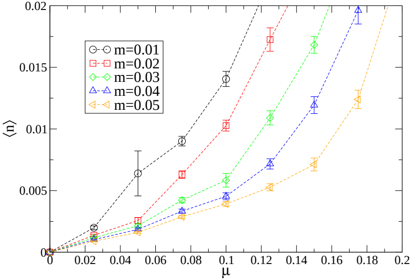

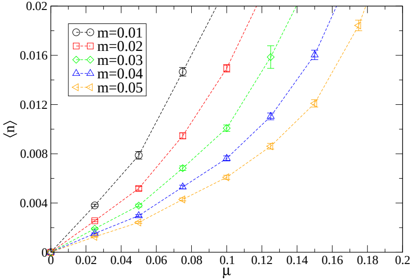

full

ph'q'

8^3

m	0.01	0.02	0.03
$M_\pi/2$	0.10(1)	0.140(2)	0.199(2)

m	0.01	0.02	0.05
M_f	0.36(3)	0.33(4)	0.48(4)

$\Rightarrow$ difference in μ_c expected

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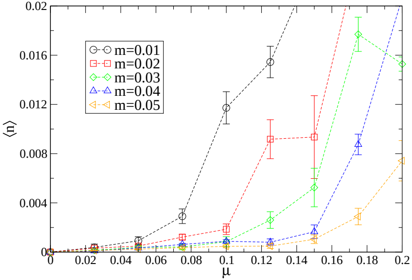

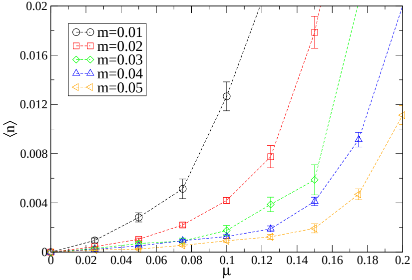

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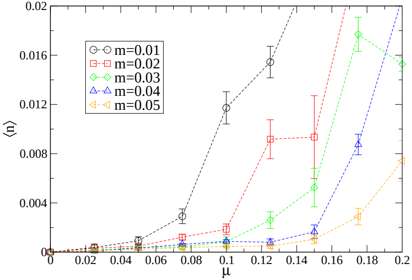

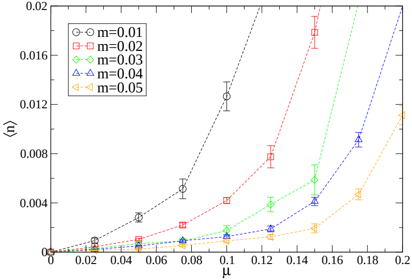

full

ph'q'

12^3

no discernible difference in μ_c found

Phase factor

- Small $\langle e^{i\phi} \rangle$ (fluctuating around zero) may provide evidence for severity of sign problem.
- Some scatter plots . . .

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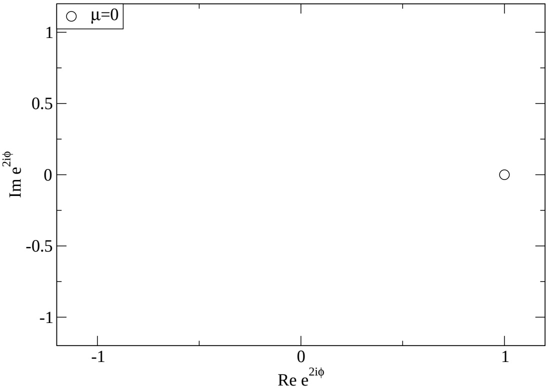
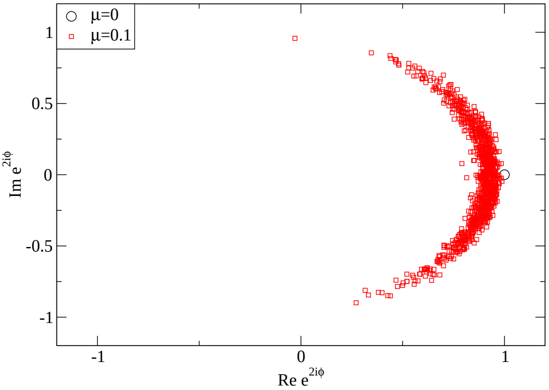


$$e^{2i\phi} = \frac{\det M(\mu)}{\det M(-\mu)}$$

Phase factor



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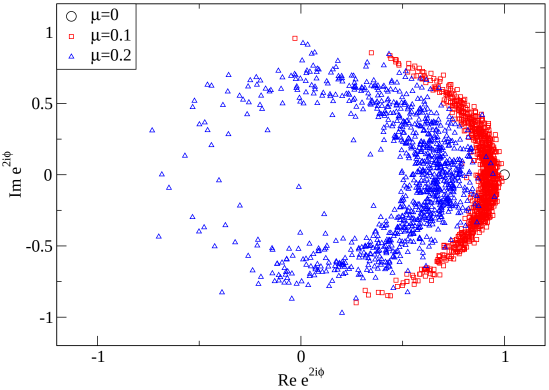

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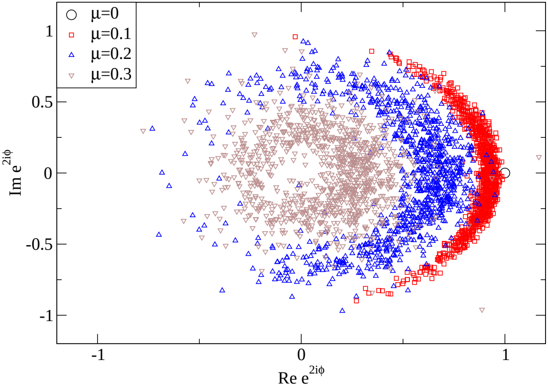

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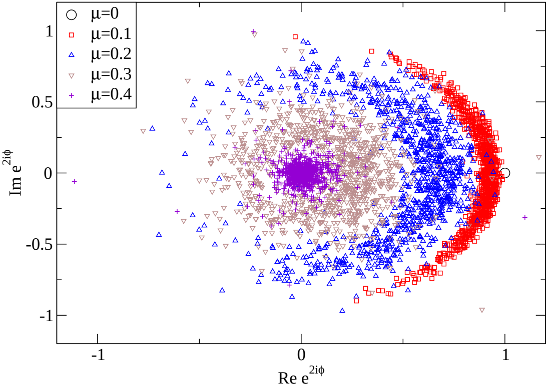

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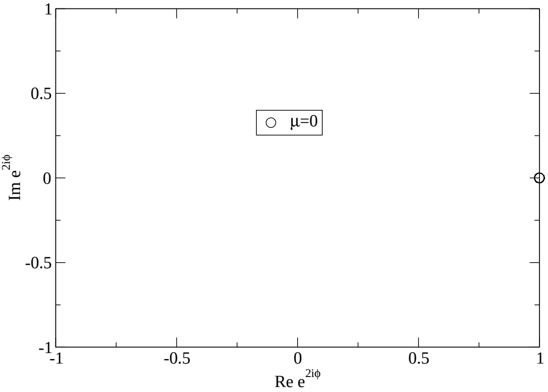

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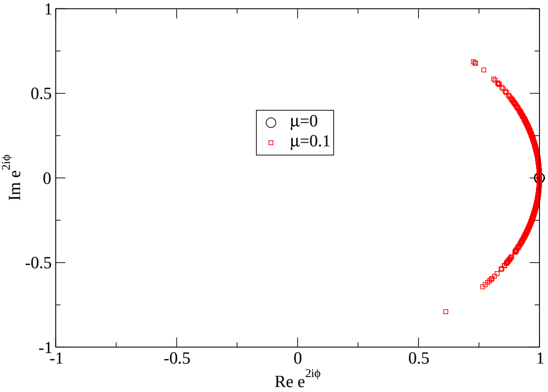

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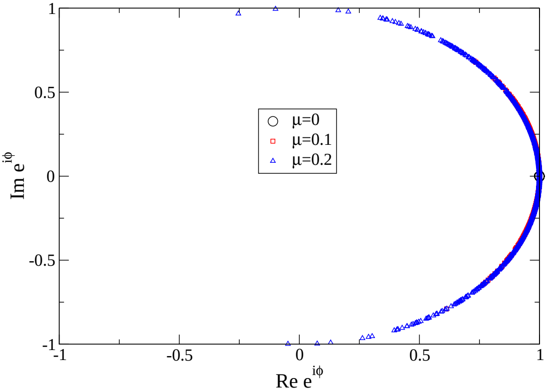

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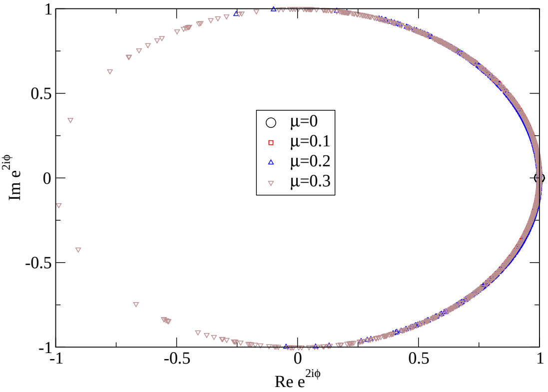

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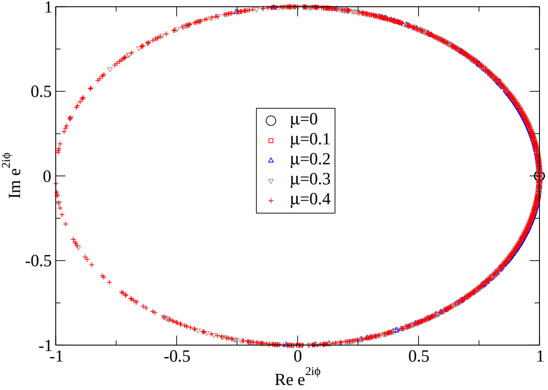

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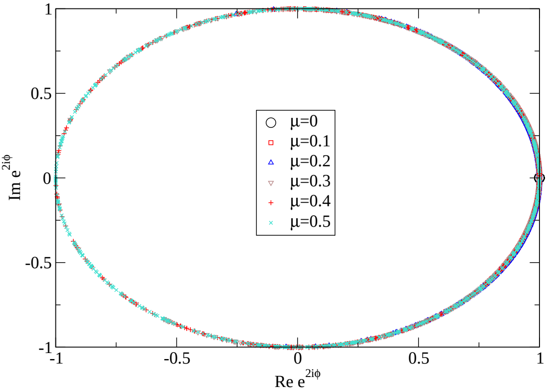

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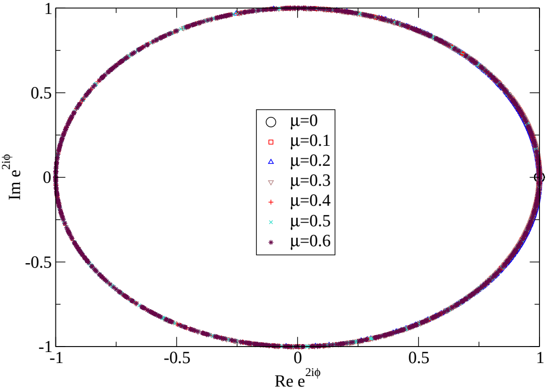

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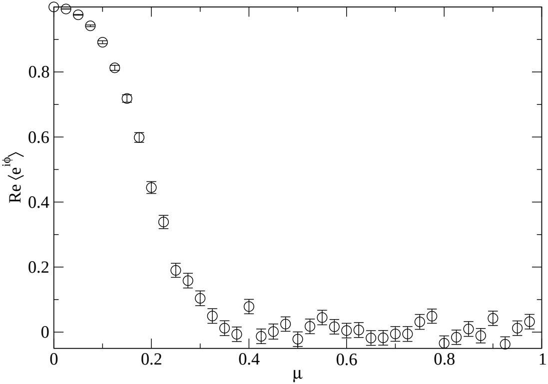

$$\left\langle e^{i\phi} \right\rangle = \left\langle \frac{\det M(\mu)}{|\det M(\mu)|} \right\rangle$$

Thirring model

So far ...

- At high μ, config's with different signs of det M sampled, but no convergence problems observed
- Complex Langevin may be well suited to handle the sign problem here

Now: compare with heavy dense limit (large m and μ)

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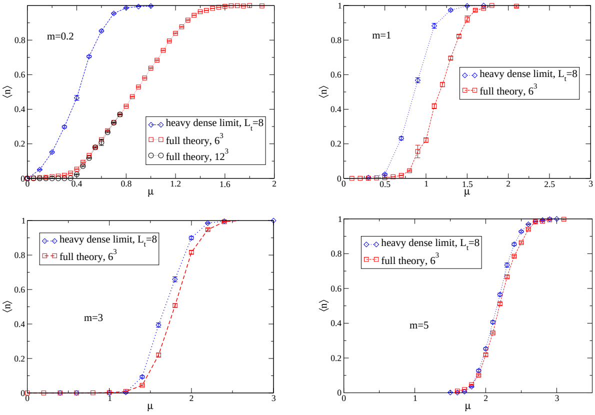
Thirring model: heavy dense limit



heavy dense limit & full theory do not agree beyond saturation

Sign problem with complex stochastic quantization: upshot for Thirring model

Complex Langevin evolution

- Sometimes successful
- Not in general reliable
  - May not converge
  - May converge against wrong solution
- No general 'a priori' criterion for success known

Reasons for hope

- Algorithm converges; no runaway trajectories
- All phases of the fermionic determinant sampled at large μ
- Observables differ between full and phase-quenched theory

Reason to be skeptical

- Onsets in full & ph'q' theories do not differ