Confinement and infrared propagators in lattice Landau gauge

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DPG spring meeting, Bonn
15 March 2010

Talk based on

- J. M. Pawlowski, D.S., I.-O. Stamatescu:
  Lattice Landau gauge with stochastic quantization
  arXiv:0911.4921 [hep-lat] (Nucl. Phys. B 830:291-314, 2010)

- A. Maas, J. M. Pawlowski, D.S., A. Sternbeck, L. von Smekal:
  Strong-coupling study of the Gribov ambiguity in lattice Landau gauge
  arXiv:0912.4203 [hep-lat] (accepted by Eur. Phys. J. C)

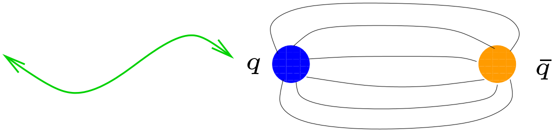
Confinement & IR propagators

Signatures of confinement

- Confining quark-antiquark potential (gauge-invariant)
  - ▶ demands an explanation
- Infrared behavior of gluon & ghost propagator (gauge-dependent)
  - ▶ here: Landau gauge, $\partial_\mu A_\mu = 0$
  - ▶ may provide an explanation
    - ★ Gribov-Zwanziger scenario
      – related to confinement via topological defects

$$\langle AA \rangle$$

$$\langle \bar{c} c \rangle$$

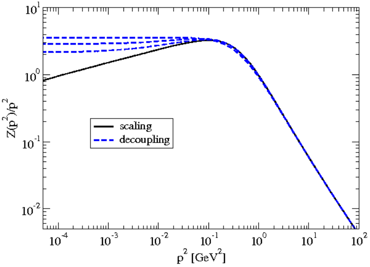


IR propagators: continuum solutions

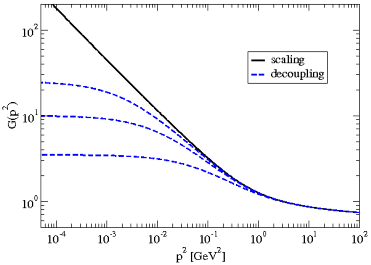
One-parameter family of solutions from FRG & DSE

(figs. from Fischer et al. '08)

Gluon propagator $D_{\rm gl}$



Ghost dressing function $q^2 D_{\rm gh}$



Infrared exponents

$$\lim_{q^2 \to 0} D_{\rm gl/gh}(q^2) \propto \frac{1}{(q^2)^{\kappa_{A/C}+1}}$$

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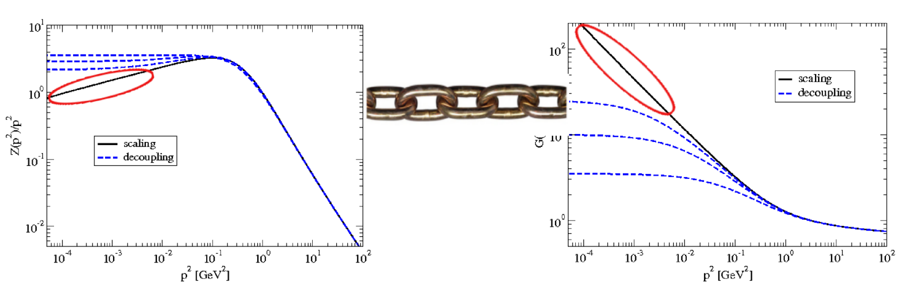

Infrared exponents

(Lerche/von Smekal '02, Zwanziger '01, Pawlowski et al. '03)

$$\lim_{q^2 \to 0} D_{\mathrm{gl/gh}}(q^2) \propto \frac{1}{(q^2)^{\kappa_{A/C}+1}}$$

Scaling: $\kappa_A = -2 \underbrace{\kappa_C}_{=:\kappa} + \frac{d-4}{2}$

IR gluon & ghost propagator

Confinement ⇒ IR behavior

- Common confinement scenarios (Gribov-Zwanziger, Kugo-Ojima) predict infrared behavior
  - G-Z/K-O $\xrightarrow[\text{global BRST}]{}$ scaling solution

Problem

- Lattice: No scaling – only decoupling ($d = 3, 4$)

IR behavior ⇒ confinement

- $\boxed{\text{Scaling solution and decoupling solutions are confining}}$ (Braun et al. '07)
- At issue:
  - $\boxed{\text{Confinement mechanism}}$
  - Status of global BRST symmetry   scaling: ✓   decoupling: ✗

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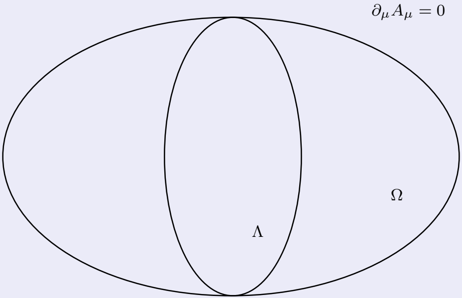
Gribov problem

Relevant regions in configuration space



- G.f. $\rightsquigarrow -\partial\mathcal{D} > 0$: 1st Gribov region Ω
- Still multiple gauge copies (Gribov '78, Singer '78)

  Gribov ambiguity
- Unique copy: Fundamental modular region Λ

  NP-hard optimization problem

Gribov problem

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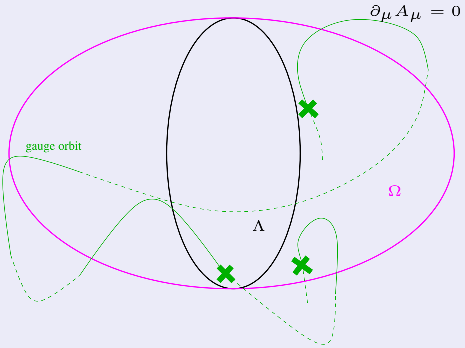

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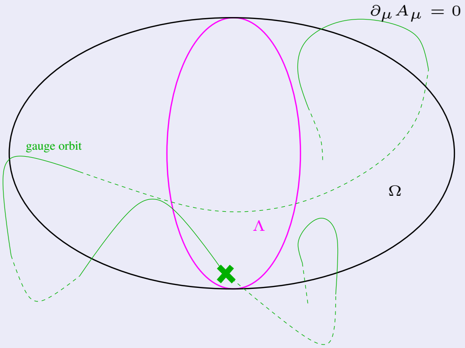

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Gauge fixing methods

Different methods employed here

1. Stochastic gauge fixing — finite β
2. Standard gauge fixing — here: $\beta = 0$
3. 'max-B' gauge — here: $\beta = 0$

	g.f. per copy	
	std.	non-std.
first copy	2.	1.
best copy	3.	

- here: 'best' $\hat{=}$ IR max. ghost
  - alt.: global max. of g.f. functional ($\rightsquigarrow$ 10% effect on ghost)

(Bornyakov et al. '08, Bogolubsky et al. '08)

- here: quenched $SU(2)$
- $N_c = 3$ (Bogolubsky et al. '07, Cucchieri et al. '07, Oliveira et al. '07, Sternbeck et al. '07) or dyn. fermions (Ilgenfritz et al. '06) $\Big\}$ not crucial

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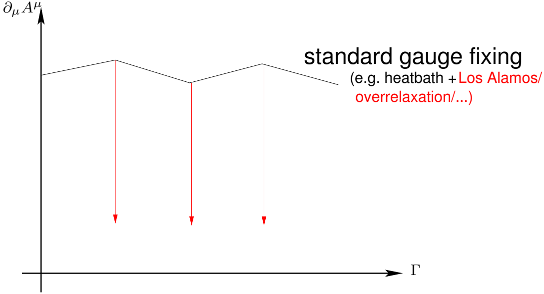
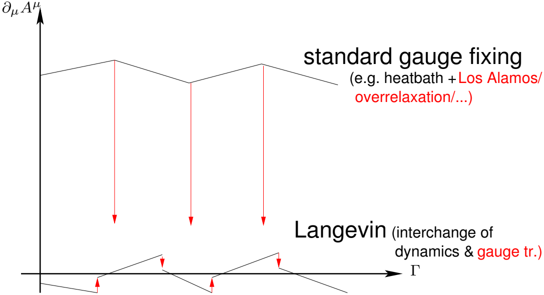
Stochastic gauge fixing: Intuitive picture



(cp. Nakamura/Saito/Sakai '04)

Stochastic gauge fixing: Intuitive picture



(cp. Nakamura/Saito/Sakai '04)

Langevin eq. incl. gauge force (Zwanziger '81)

more local than standard g. f.

$\Rightarrow$ possibly different sampling of config' space

Propagators from stochastic gauge fixing

Bottom line

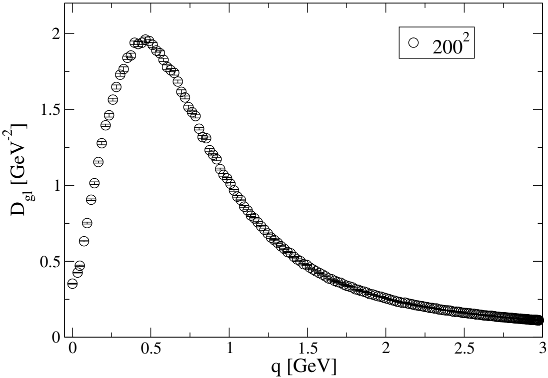
Confirmation of
standard lattice results . . .

2D: scaling, 3&4D: decoupling
e.g.

- 2D: Maas '07
- 3D: Cucchieri/Mendes '03
- 4D: Sternbeck et al. '05, '06, Bogolubsky et al. '07, '08, '09, Cucchieri/Mendes '07 . . .

. . . with alternative
gauge fixing method

Propagators from stochastic gauge fixing



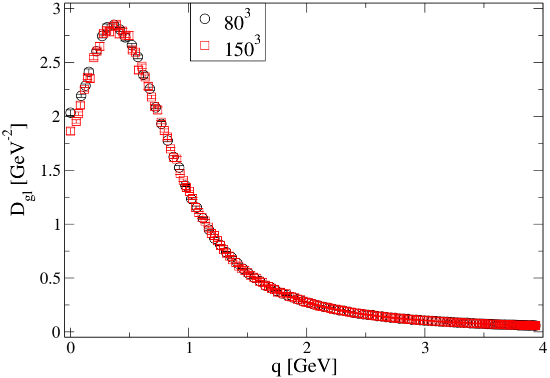
Gluon propagator

2D: compatible with IR scaling

- $\kappa \approx 0.2$, as expected

But: $\beta = 0$ results raise new questions

Propagators from stochastic gauge fixing



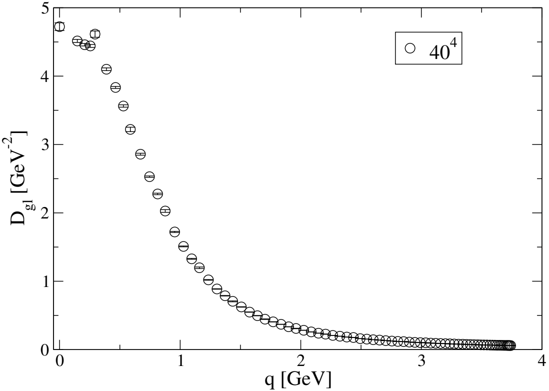
Gluon propagator

3D: Peak, but IR finite

$\rightsquigarrow$ decoupling

But: at $\beta = 0$, 3D similar to 2D

Propagators from stochastic gauge fixing

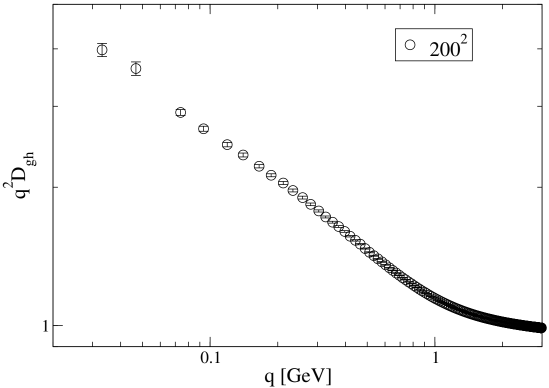


Gluon propagator

4D: IR finite

$\rightsquigarrow$ decoupling

Propagators from stochastic gauge fixing



Ghost dressing function

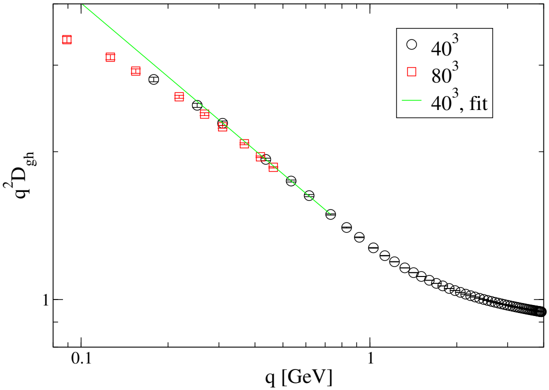
2D: IR divergent

$\kappa \equiv \kappa_C \approx 0.2$

- as expected for scaling
- consistent with gluon data

But: different picture at $\beta = 0$

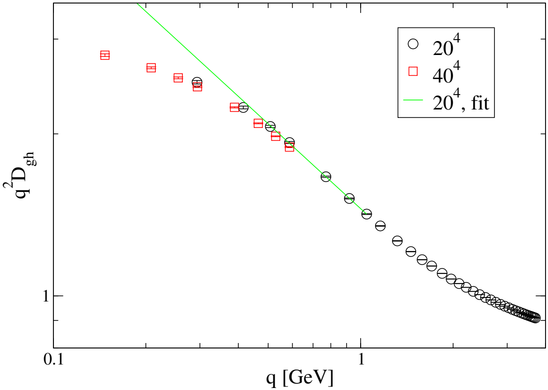
Propagators from stochastic gauge fixing



Ghost dressing function

3D: tends towards decoupling

Propagators from stochastic gauge fixing



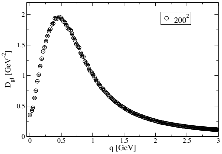
Ghost dressing function

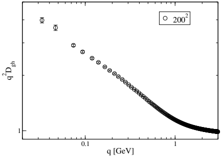
4D: tends towards decoupling

Upshot of stochastic gauge fixing

gluon | ghost | scaling

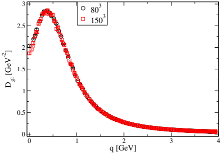
$d = 2$

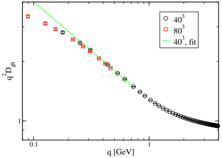




✓

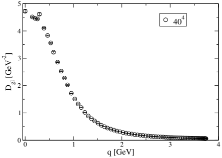
$d = 3$

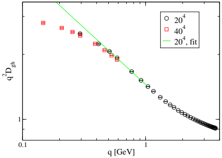




✗

$d = 4$





✗

Upshot of stochastic gauge fixing

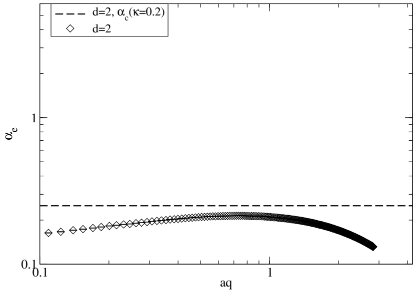
- Confirmation of standard lattice scenario
- $\Rightarrow$ Lattice gauge fixing problem more general?

Strong-coupling limit

Motivation

- $$\beta \to 0 \qquad \hat{=} \qquad a \to \infty$$
  IR behavior visible at large aq
- 4D: Evidence for conformal behavior (Sternbeck/von Smekal '08)
- Recent new data also in $d = 2$ & 3
  Interpretation under debate (Cucchieri/Mendes '09, Maas et al. '09)

Running coupling



cp. with continuum predictions for α_c

(Lerche/von Smekal '02)

$$\text{peak at} \quad \left.\begin{matrix} 85\% \\ \\ \\ \end{matrix}\right\} \text{ of } \alpha_c \text{ in } d = \left\{\begin{matrix} 2 \\ \\ \\ \end{matrix}\right.$$

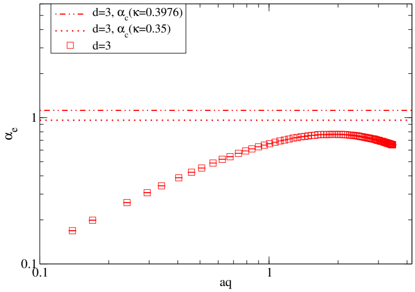
eff. running coupling

$$\alpha_e = \frac{g^2}{4\pi} q^{d+2} D_{\text{gl}} D_{\text{gh}}^2$$

here

- strong-coupling limit
- standard gauge fixing

Running coupling



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peak at 70% $\Big\}$ of α_c in $d = \Big\{ 3$

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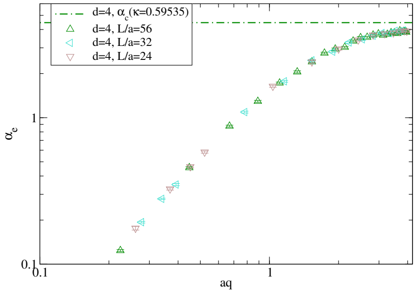

cp. with continuum predictions for α_c

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peak at

90%

of α_c in $d =$

4

← 4d data: Sternbeck/von Smekal '08

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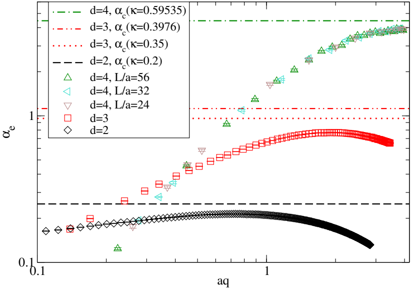

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$$\text{peak at} \begin{array}{c} 85\% \\ 70\% \\ 90\% \end{array} \Bigg\} \text{ of } \alpha_c \text{ in } d = \begin{cases} 2 \\ 3 \\ 4 \end{cases}$$

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Upshot of standard gauge fixing at $\beta = 0$

$d = 2$, 3 and 4

- no uniform scaling at all aq
- possible scaling window
  - moves towards larger aq from $d = 2$ to $d = 4$
  - same pattern at finite β
- previous result: discretization effects at small aq (Sternbeck/von Smekal '08)

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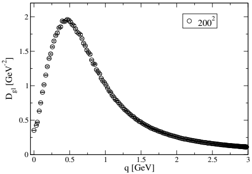
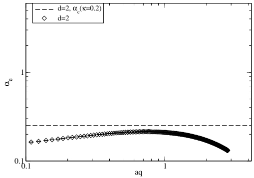
Room for speculation . . .

gluon prop., finite β | running coupling, $\beta = 0$

$d = 2$





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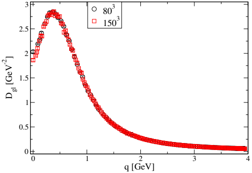

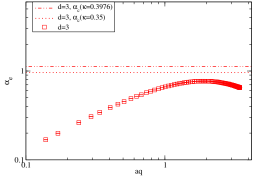

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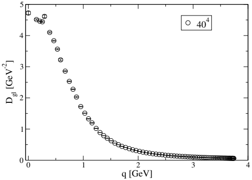

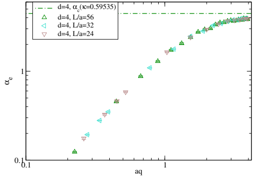

Impact of Gribov ambiguity

Basic idea of 'Landau max-B' gauge (Maas '09)

- Functional methods $\rightsquigarrow$ $\boxed{\text{one-parameter}}$ family of solutions

  (Fischer et al. '08, Boucaud et al. '08)

- Analogous lattice procedure
  - Exploit residual gauge freedom in Landau gauge
  - For each config'
    - choose among n_{copy} Gribov copies the one with

$$\frac{D_{\text{gh}}(q_{\text{min}})}{D_{\text{gh}}(Q)} =: \boxed{B} = \max$$

- Here: first simulations at $\beta = 0$, in $d = 2, 3$

  finite β: Maas '09 & work in progress

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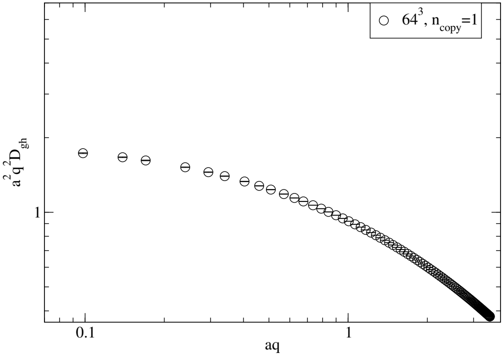
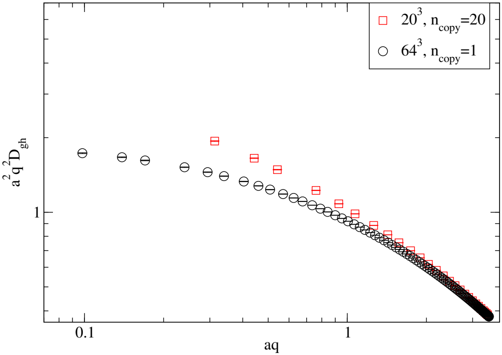


$d = 3$: ghost dressing function

- increase n_{copy} ...

  $\rightsquigarrow$ strong effect

Impact of Gribov ambiguity



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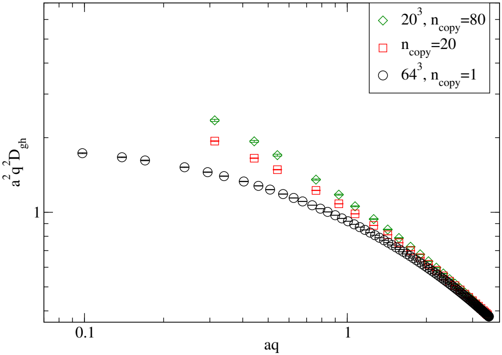

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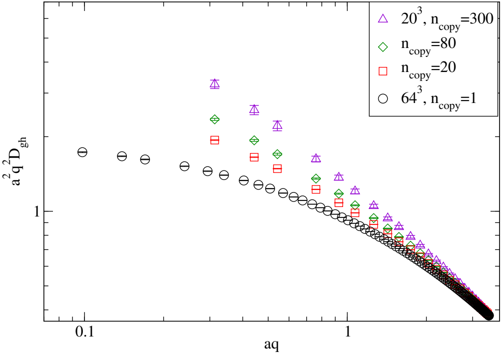

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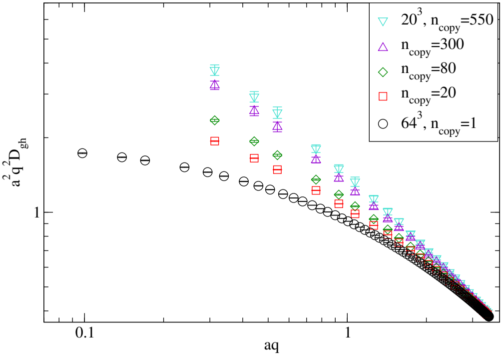

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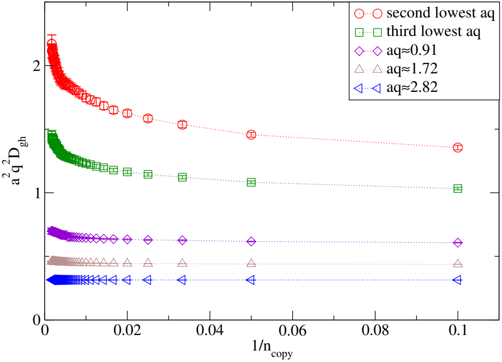
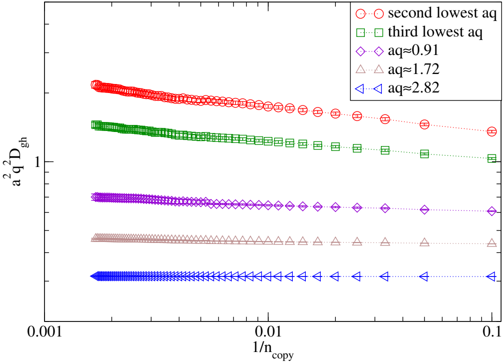


$d = 2$: ghost dressing function vs. n_{copy}^{-1}

- No 'saturation' at 600 copies

$\Rightarrow$ No $n_{copy} \to \infty$ extrapolation possible

Impact of Gribov ambiguity

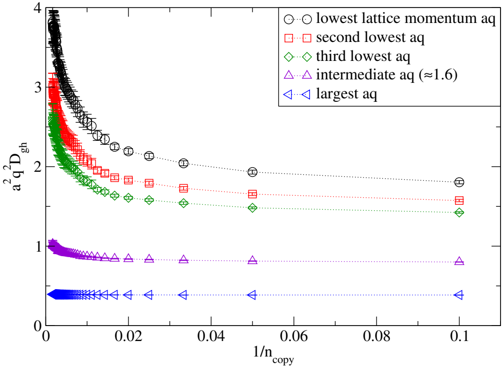


$d = 2$: ghost dressing function vs. n_{copy}^{-1}

- No 'saturation' at 600 copies (log-log plot)

$\Rightarrow$ No $n_{\text{copy}} \to \infty$ extrapolation possible

Impact of Gribov ambiguity



$d = 3$: ghost dressing function vs. n_{copy}^{-1}

- No 'saturation' at 600 copies

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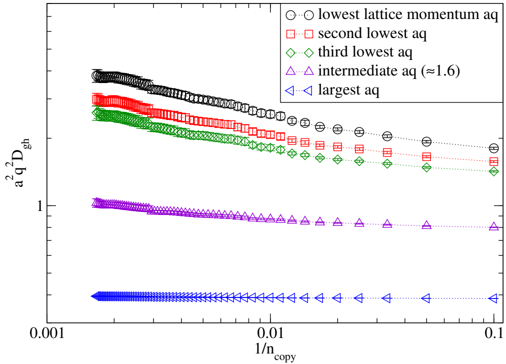
Impact of Gribov ambiguity



$d = 3$: ghost dressing function vs. n_{copy}^{-1}

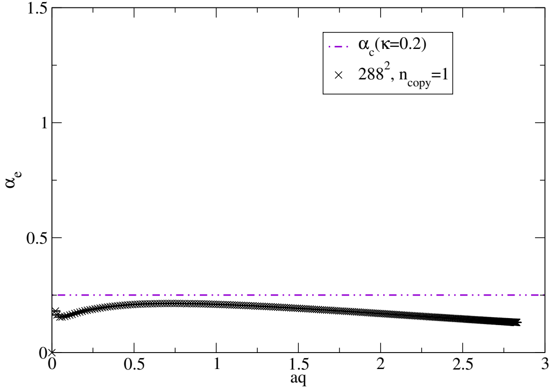
- No 'saturation' at 600 copies (log-log plot)

  $\Rightarrow$ No $n_{copy} \to \infty$ extrapolation possible

Impact of Gribov ambiguity on running coupling

- $\alpha_e \propto D_{\mathrm{gh}}^2 D_{\mathrm{gl}}$
- Large impact on ghost
- Small impact on gluon

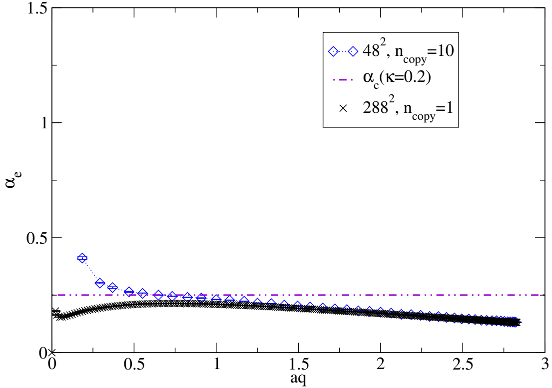
Impact of Gribov ambiguity on running coupling



$d = 2$

Large Gribov copy effect on running coupling

Impact of Gribov ambiguity on running coupling



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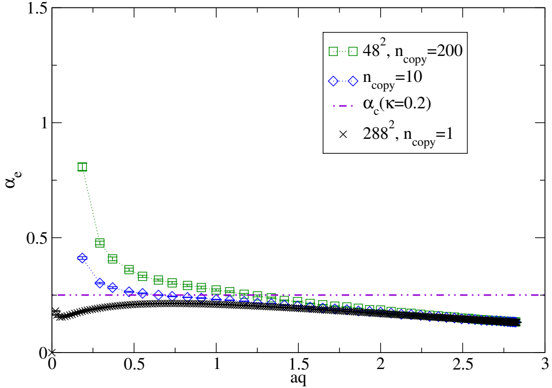

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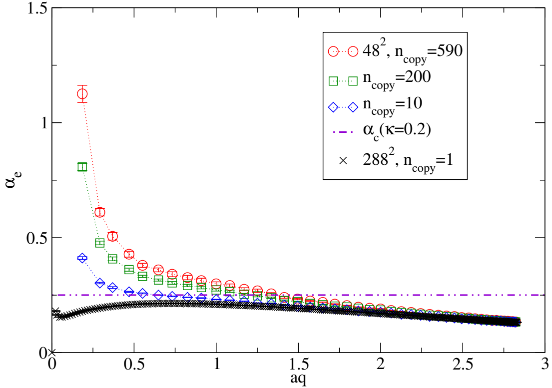

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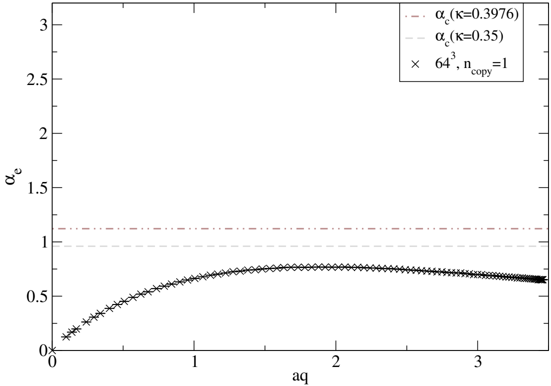

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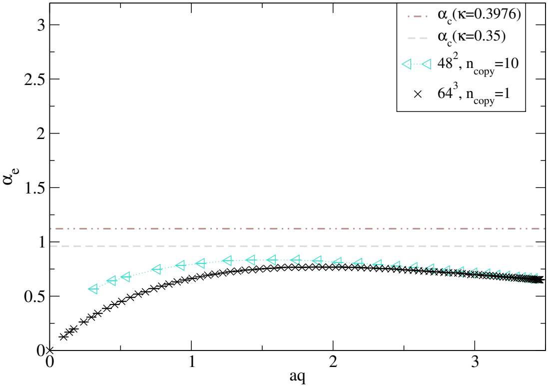

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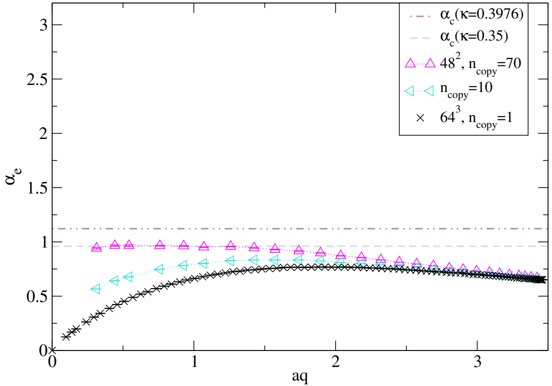

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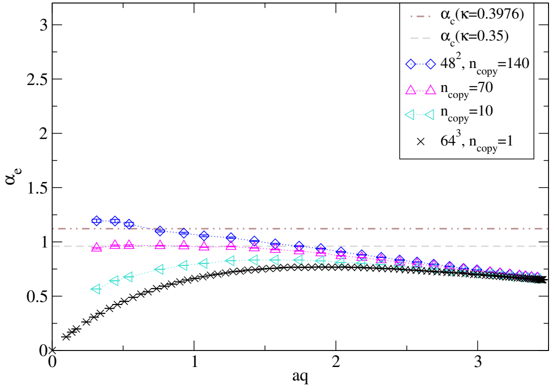

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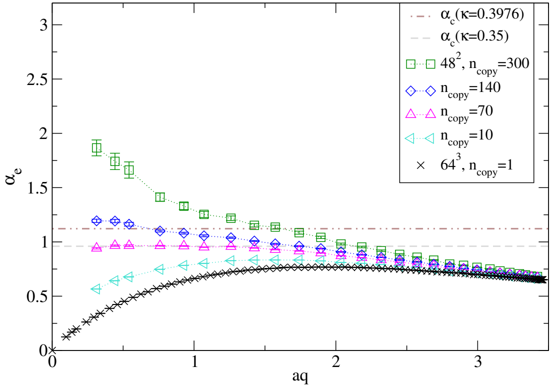

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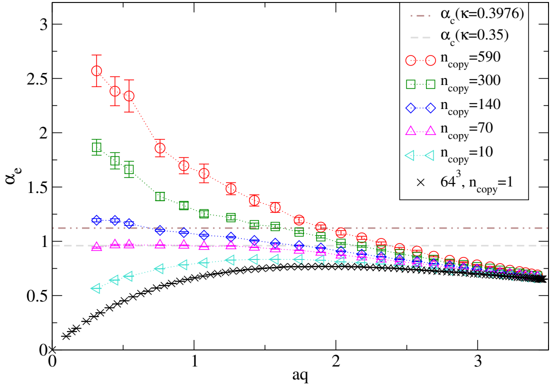


$d = 3$

Large Gribov copy effect on running coupling

$\Rightarrow$ even 'over-scaling'

Impact of Gribov ambiguity

Interpretation

- Gribov ambiguity more severe than expected
  - Should also hold at finite β
- No subset of IR solutions (scaling & decoupling) excluded by our data
  - Stronger & diff. Gribov copy effect than by 'global maximization'

    (e.g. Bogolubsky et al. '05, '07, '09, Bornyakov et al. '08, '09)

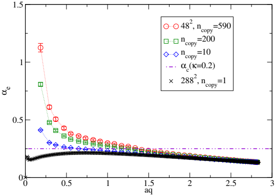
    - ★ $\mathcal{O}$ (100%) vs. 10% for ghost

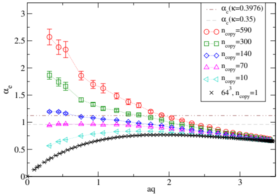
      gluon: small effect vs. 20%

    - ★ 'over-scaling' vs. decoupling

      but: 'over-scaling' ruled out in continuum

      (uniqueness proof: Fischer et al. '06, '09, Huber et al. '07)





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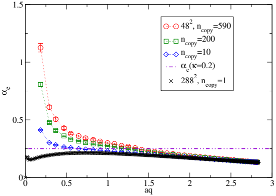

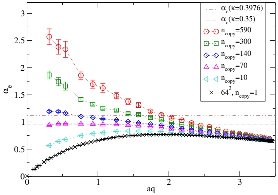

Summary

Stochastic gauge fixing

- Standard lattice scenario supported by alternative gauge fixing method

Strong-coupling limit

- Evidence against scaling (in agreement with std. lattice scenario)
  – but also in $d = 2$ (surprising)

Max-B gauge in strong-coupling limit

- Unexpectedly strong effect of Gribov ambiguity
  - Surprising 'over-scaling' in finite volumes
  - No subset of solutions can be excluded yet

Outlook

- Current results surprise and raise new questions
  - $V \to \infty$ and $n_{\text{copy}} \to \infty$ limit of max-B gauge
  - Max-B gauge at finite β (Maas '09 & work in progress)
- BRST invariant lattice formulation . . . (von Smekal et al. '07, '08 . . .)

Langevin dynamics (stochastic quantization: Parisi/Wu '81, Batrouni et al. '85, Damgaard/Hüffel '87)

- Langevin equation in fictitious time θ

$$dA = (-\nabla S + \eta)\, d\theta$$

$\eta =$ Gaussian random noise

$$\xrightarrow[\text{equilibrium}]{} \quad \rho[A] \propto e^{-S[A]}$$

Langevin dynamics with gauge fixing (continuum)

- Supplement Langevin equation by gauge force (Zwanziger '81)

$$dA = (-\nabla S + \mathcal{D}v + \eta)d\theta$$

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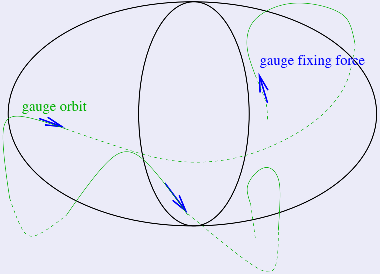
$$\xrightarrow[\text{equilibrium}]{} \quad \rho[A] \propto e^{-S[A]} \quad \& \quad \partial_\mu A_\mu = 0$$



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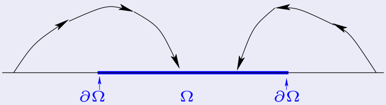
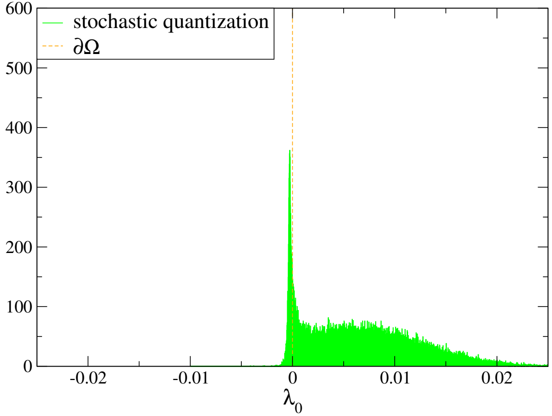
- Inside Ω: restoring towards gauge fixing surface

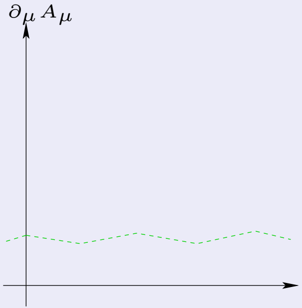
outside Ω: repulsive directions



Faddeev-Popov operator spectrum



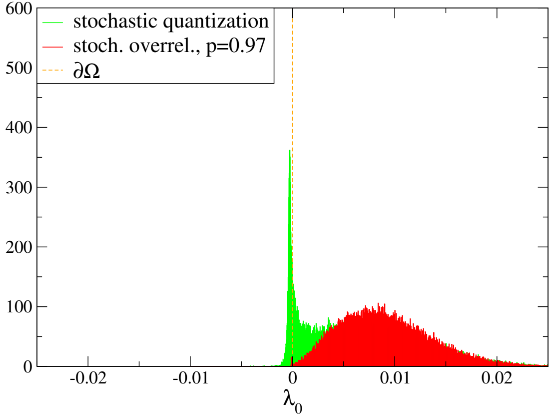
imperfect g. f.



$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$
by SQ

$\Delta^a \hat{=} \partial_\mu A^a_\mu$

Faddeev-Popov operator spectrum



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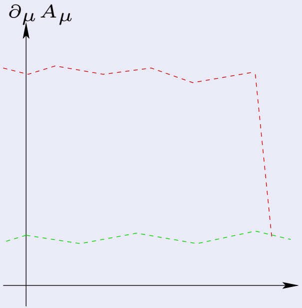
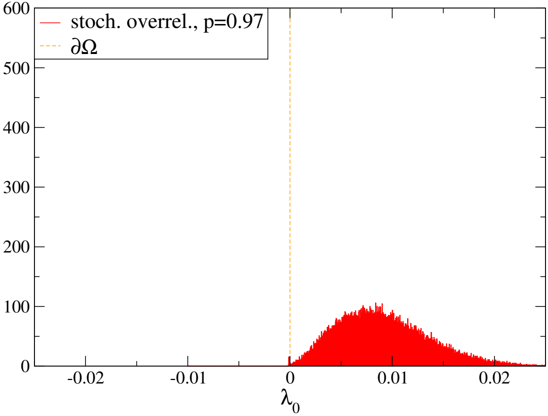


$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$

by SQ vs. stoch. overrel.

after heatbath

Faddeev-Popov operator spectrum



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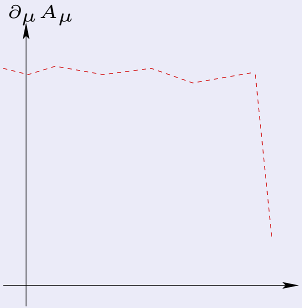
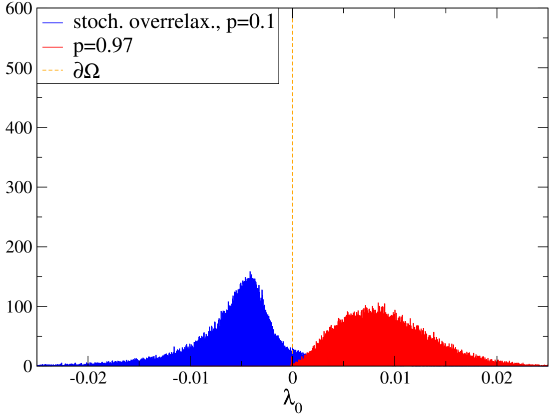


$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$
by stoch. overrel. after heatbath at different overrel. parameters

Faddeev-Popov operator spectrum



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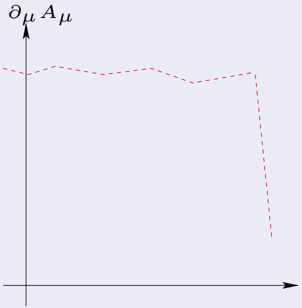

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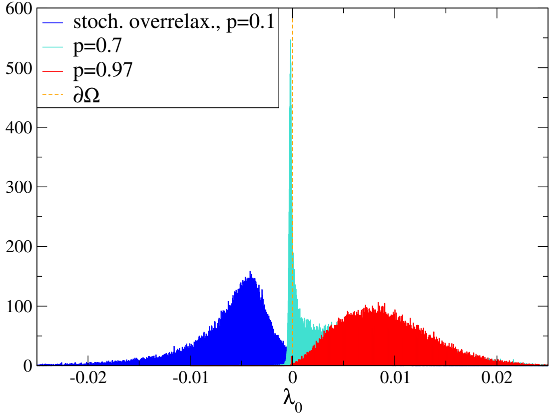

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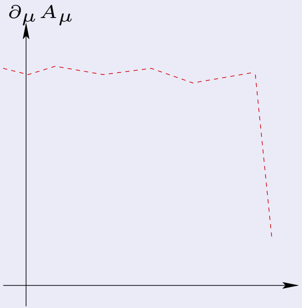

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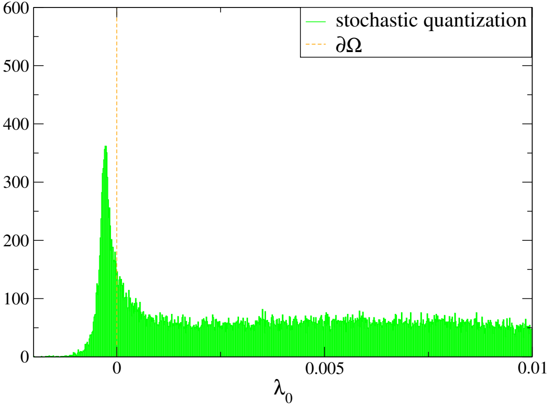

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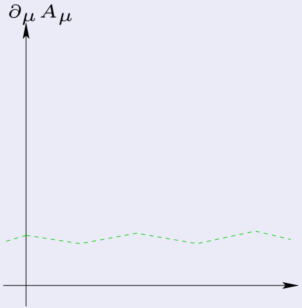

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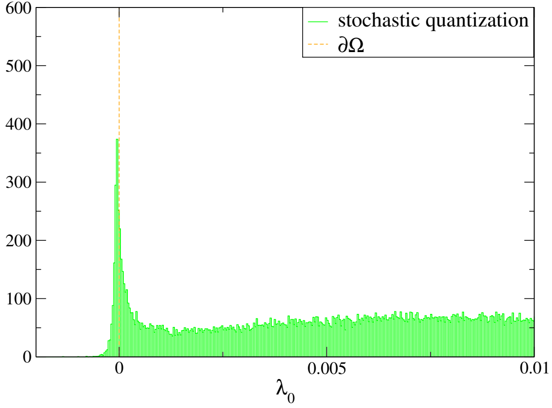
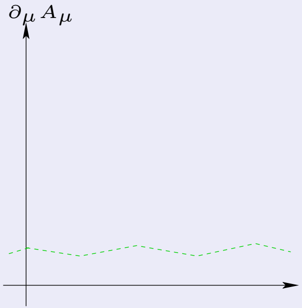
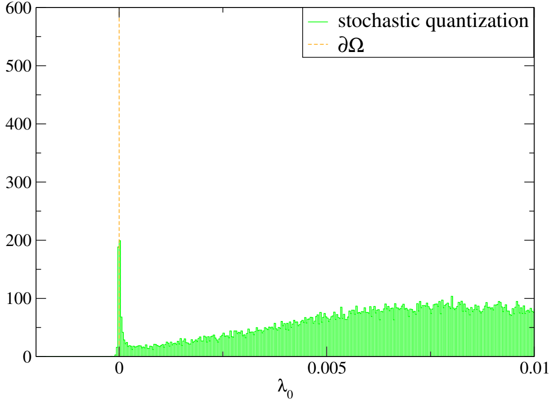


$\Delta^2 \longrightarrow 0$



$\Delta^2 \approx 3 \cdot 10^{-5}$
by SQ

Faddeev-Popov operator spectrum



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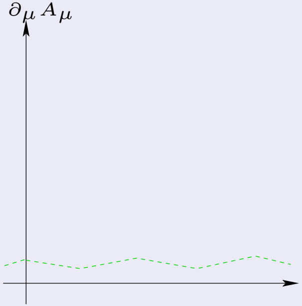
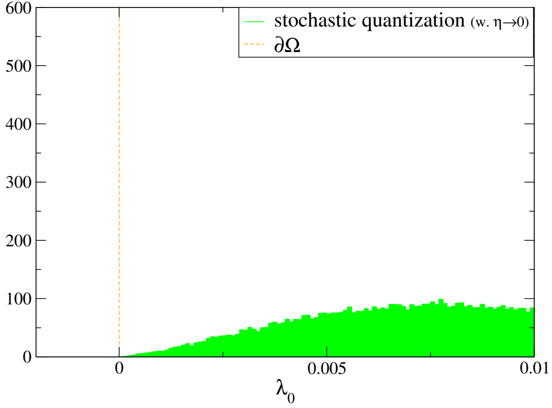
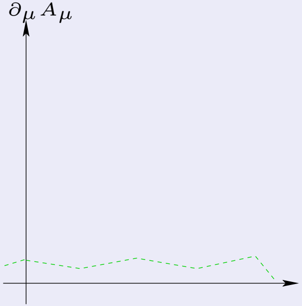


$\Delta^2 \approx 1.5 \cdot 10^{-6}$
by SQ

Faddeev-Popov operator spectrum



$\Delta^2 \to 0$



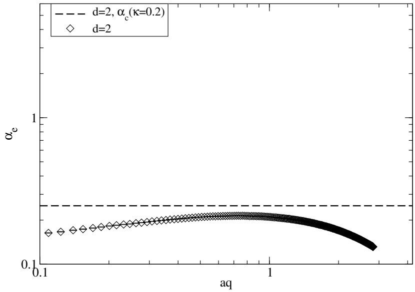
$\Delta^2 < 10^{-15}$
by SQ & step size $\to 0$

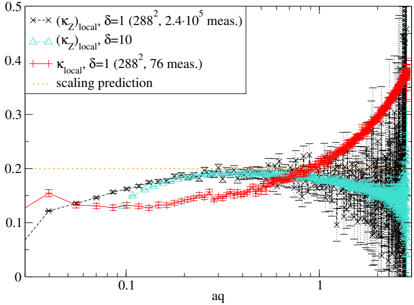
Faddeev-Popov operator spectrum

Upshot

Peak disappears for sufficiently small Δ^2

Running coupling & local exponents



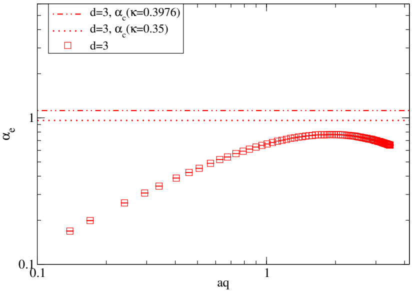


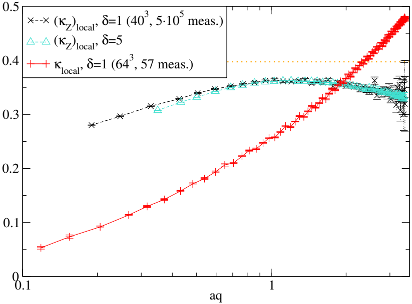
cp. with continuum predictions for α_c

(Lerche/von Smekal '02)

$$\text{peak at} \left.\begin{matrix} 85\% \\ \\ \end{matrix}\right\} \text{ of } \alpha_c \text{ in } d = \begin{cases} 2 \\ \\ \end{cases}$$

Running coupling & local exponents



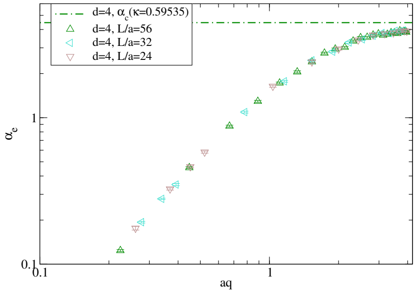


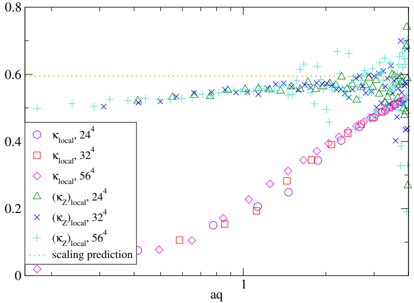
cp. with continuum predictions for α_c

(Lerche/von Smekal '02)

$$\text{peak at } 70\% \left.\vphantom{\begin{matrix}a\\b\\c\end{matrix}}\right\} \text{ of } \alpha_c \text{ in } d = \left\{\vphantom{\begin{matrix}a\\b\\c\end{matrix}} 3 \right.$$

Running coupling & local exponents





cp. with continuum predictions for α_c

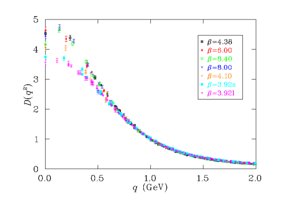
(Lerche/von Smekal '02)

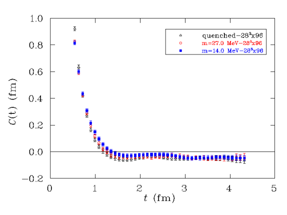
$$\left.\begin{array}{l}\text{peak at} \\ \qquad 90\% \end{array}\right\} \text{ of } \alpha_c \text{ in } d = \left\{\begin{array}{l} \\ 4 \end{array}\right.$$

Importance of IR asymptotics

What κ may tell about confinement

- IR asymptotics of $D_{\mathrm{gl}}(q^2)$, $D_{\mathrm{gh}}(q^2)$
  $\rightsquigarrow$ Test predictions of confinement scenarios
- E.g.: Violation of reflection positivity by IR non-divergent gluon propagator (lattice: Bowman et al. '07 (4D), Cucchieri et al. '05 (3D))

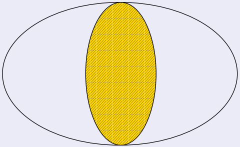




How to sample the configuration space

Possible requirements on the algorithm

Find either of the following:



- Fundamental modular region Λ (free of Gribov copies) $\hat{=}$ NP-hard optimization problem
- Entire Gribov region Ω with Faddeev-Popov weight
  - (Conjecture: Ω & Λ equiv. for expectation values (Zwanziger '04))
  - Restriction to Ω automatically (by g.f.) precise distribution nontrivial
- Bias towards Gribov horizon $\partial\Omega$ (Gribov-Zwanziger sc.: config's near $\partial\Omega \cap \partial\Lambda$ account for confinement)

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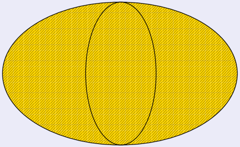

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  - Restriction to Ω automatically (by g.f.)
    - ★ Faddeev-Popov operator is Hessian of $\int d^4x |{}^{\omega}A|^2$, which is minimized by numerical Landau g. f.;

  precise distribution nontrivial
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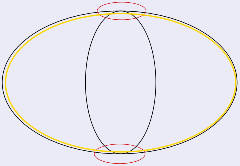

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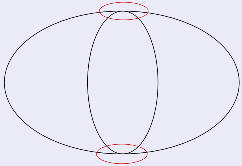
Confinement scenarios

Gribov-Zwanziger scenario (Gribov '78, Zwanziger '94, '04)

- Basic idea: Configurations in vicinity of $\partial\Omega(\cap\partial\Lambda)$ account for confinement of gluons.



  - Entropy favors $\partial\Omega$ (due to $r^{N-1}dr$).
  - (Situation less clear due to e^{-S})
- Implications depend on gauge...

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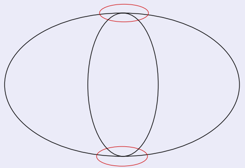
- Basic idea: Configurations in vicinity of $\partial\Omega(\cap\partial\Lambda)$ account for confinement of gluons.



  - Entropy favors $\partial\Omega$ (due to $r^{N-1}dr$).
  - (Situation less clear due to e^{-S})
- Implications depend on gauge...
  - In Landau gauge:
    IR vanishing of gluon propagator (horizon condition).
    ('IR slavery' scenario of *quark* confinement, $D \sim 1/q^4$, obsolete.)

Confinement scenarios

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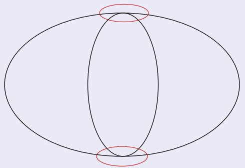

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Confinement scenarios

Kugo-Ojima scenario (Kugo/Ojima '79, review: Nakanishi/Ojima '90)

- Confinement by BRST quartet mechanism
  - $\approx$ Gupta-Bleuler in QED, but applies also to *transversal* gluons
- Integral part of Kugo-Ojima criterion:
  Well-defined global color charge

  $\Longleftrightarrow$

  1. mass gap &
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Confinement scenarios and Green's functions

Implications in a nutshell

- Kugo-Ojima
  - (BRST quartet mechanism, $\approx$ Gupta-Bleuler in QED)

  $\kappa > 0$
- Gribov-Zwanziger (stronger implications)
  - (confinement by config's close to 1st Gribov horizon)

  $\boxed{\kappa > 0}$ for ghost, $\kappa > 1/2$ for gluon (i.e. $\boxed{\kappa_A < -1}$)

  I.e.:
  - Ghost dressing function IR divergent
  - Gluon propagator IR vanishing

Implementing stochastic gauge fixing on the lattice

Dynamics

Lattice Langevin eq. with step size ε (e.g. Batrouni et al. '85)

$$\exp\left(\ldots A_\mu^a(x)\right) = U_\mu(x) \rightarrow \exp(i\sigma^a F_{\mu x}^a) U_\mu(x)$$

$$\text{with force } F_{\mu x}^a = \underbrace{i\varepsilon \nabla_{x\mu a} S[U]}_{\text{drift}} + \underbrace{\sqrt{\varepsilon}\eta_{x\mu a}}_{\text{diffusion}} \qquad \text{`}\langle \eta_\alpha \rangle = 0,\ \langle \eta_\alpha \eta_\beta \rangle = 2\delta_{\alpha\beta}\text{'}$$

Gauge fixing

. . . via Zwanziger's drift term (lattice version: Rossi et al. '88)

$$\boxed{\Omega(x) = \exp\left(\ldots \cdot \sigma^a \frac{\Delta^a}{\alpha}\right)}$$

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general lattice g.t.: $U_\mu(x) \rightarrow \Omega(x)U_\mu(x)\Omega^\dagger(x+\hat{\mu})$

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$\Delta^a \hat{=} \partial_\mu A^a_\mu$, α: gauge parameter

Implementing stochastic gauge fixing

Random walk

For dynamics: Alternatively to Langevin:
Random walk with step size $\eta \in \mathcal{O}(\sqrt{\varepsilon})$

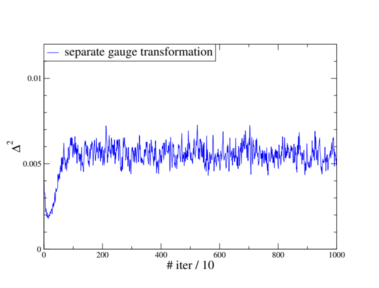
$$U_\mu(x) \rightarrow \exp(i\sigma^a A_\mu^a) U_\mu(x)$$

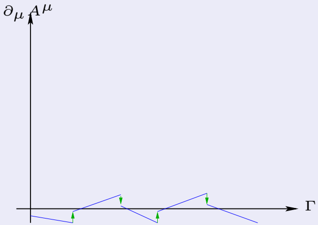
with contributions $\pm\eta$ (fixed size) to $A_\mu^a(x)$,
accepted with probability

$$p = \frac{1}{2}\left(1 \pm \tanh\left(\frac{1}{2}\eta i \nabla_{x\mu a} S[U]\right)\right)$$

Implementing stochastic gauge fixing

- Langevin vs. random walk: No important difference
- 'Local' interchange of dynamic updates and gauge transf's
  $\Rightarrow$ config's closer to g. f. surface





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