Confinement in Sp(2) lattice gauge theory

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- Finite-temperature investigations

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- Aim of this work: Identify the d.o.f. that are responsible for confinement in $Sp(2)$ gauge theory (neglecting fermions for simplicity).
- Method of this work: Lattice gauge theory.
    - Crucial ingredient: Gauge fixing before performing projections to subgroups.

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  - Characterization of $U \in Sp(N,\mathbb{C})$ in $2N$-dim. rep.:

$$\begin{pmatrix} 0 & \mathbb{1}_{N\times N} \\ -\mathbb{1}_{N\times N} & 0 \end{pmatrix} \equiv J = U^T J U$$

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- Def. of $Sp(N)$:
  - Symplectic groups constitute one of four types of classical Lie groups.
  - Alternative def.: $Sp(N)$ as group of transformations which preserve the inner product

$$\langle u, v\rangle = \bar{u}_1 v_1 + \ldots + \bar{u}_N v_N \quad (u, v \in \mathbb{H}^N)$$

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  - Moreover: $\pi_1(Sp(N)/\mathbb{Z}_2) = \mathbb{Z}_2$.
  - $\Rightarrow$ Possible test of the center vortex mechanism of confinement

Introduction

Parametrization of the 4D fund. rep. of $Sp(2)$:

- Our choice:

$$Sp(2) \ni g = \begin{pmatrix} \cos\theta\, h_1 & \sin\theta\, \bar{h}_2 h_3 \\ -\sin\theta\, h_2 h_1 & \cos\theta\, h_3 \end{pmatrix}$$

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- Alternatively:

$$g = \begin{pmatrix} W & X \\ -X^* & W^* \end{pmatrix} \text{ where } W, X \in \mathbb{C}^{2\times 2}$$

and $(WW^\dagger + XX^\dagger = \mathbb{1}) \wedge (WX^T = XW^T)$

Confinement in $Sp(2)$

Static quark potential

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Static quark potential

- $\hat{V}(\hat{R}) = -\lim_{\hat{T}\to\infty} \frac{1}{\hat{T}} \ln W(\hat{R},\hat{T})$
  $\hat{V} = \sigma a^2 \hat{R} - \alpha/\hat{R} + c$

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- Algorithm: Cabibbo-Marinari

  + microcanonical reflections

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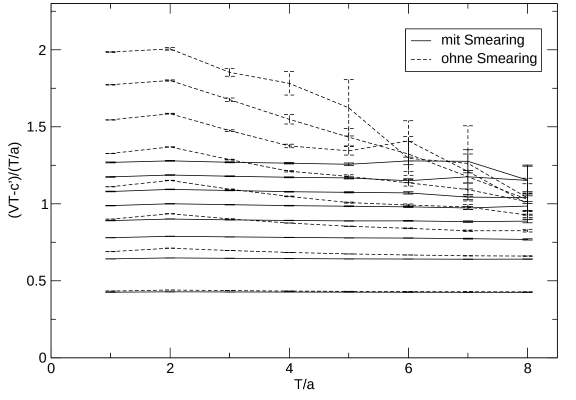
- Algorithm: Cabibbo-Marinari

  \+ microcanonical reflections

  \+ random GT

  \+ Smearing
  $(U_i \longrightarrow \prod_a \exp(\vartheta_a C_a) U_i$,
  $\vartheta_a \in \mathcal{O}(\varepsilon)$ s. t. $\langle U_{ij} \rangle \longrightarrow 1$,
  adjust $\varepsilon \rightsquigarrow$ acc. rate ≈ 0.5)
  ... renders $-\ln W(\hat{T})$ linear



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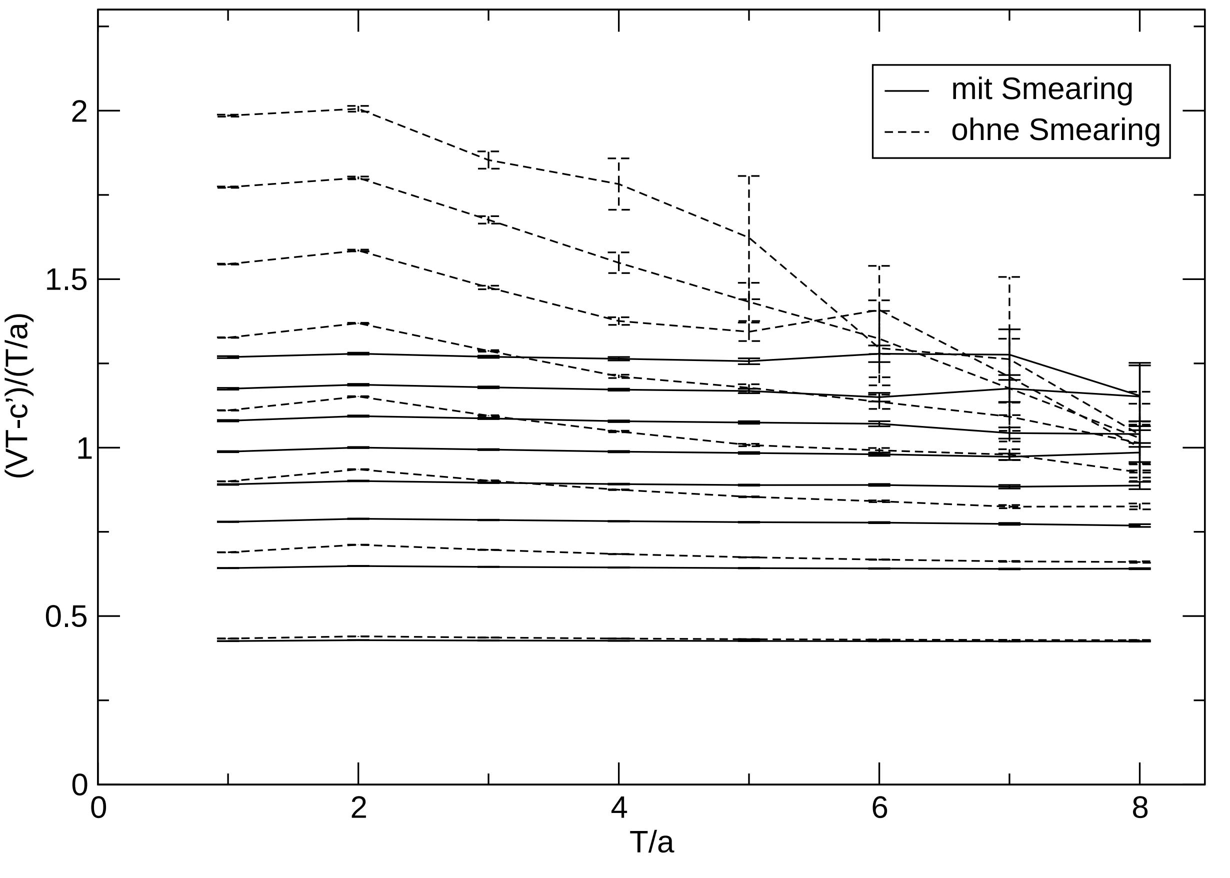
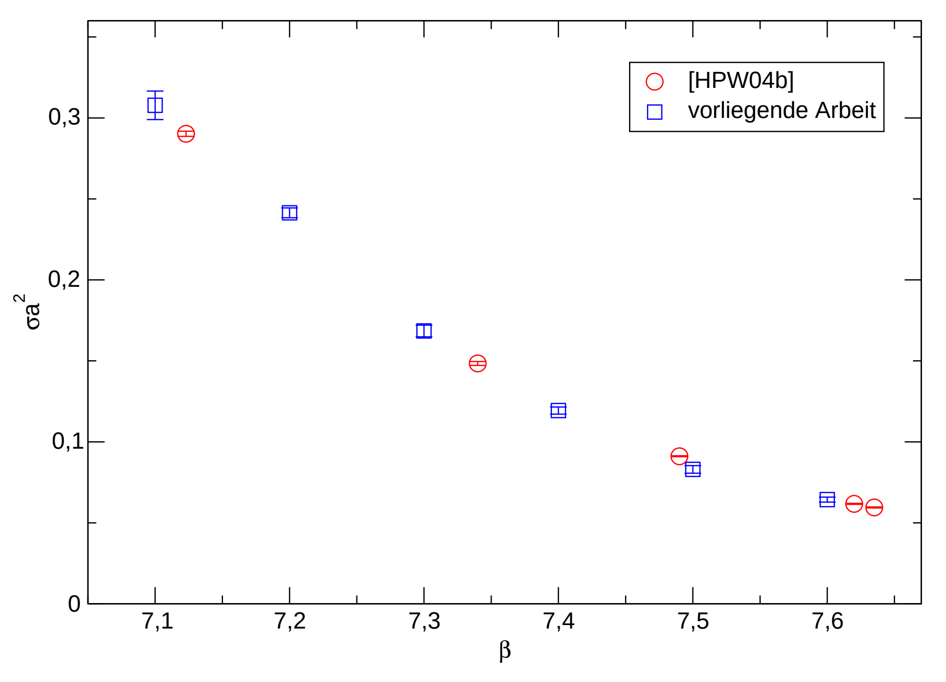


Confinement in $Sp(2)$

Static quark potential

- Agreement with previous results obtained from $\langle \Phi_{\vec{x}} \Phi_{\vec{y}} \rangle = \exp(N_\tau V(\vec{x} - \vec{y}))$ (HOLLAND, PEPE, WIESE NPB (PS) 129 ('04))



Finite temperature

Implementation:

- $T = (N_\tau a(\beta))^{-1}$

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Polyakov loop:

- Worldline of a static quark propagating in periodic time direction
- Polyakov loop on the lattice:

$$P(\vec{x}) = \prod_{t=0}^{N_\tau - 1} U_0(\vec{x}, t), \qquad \Phi(\vec{x}) = \frac{1}{N} \mathrm{tr}\, P(\vec{x})$$

- Order parameter in finite volumes:

$$\Phi = \frac{1}{L^3} \left| \sum_{\vec{x}} \Phi(\vec{x}) \right|$$

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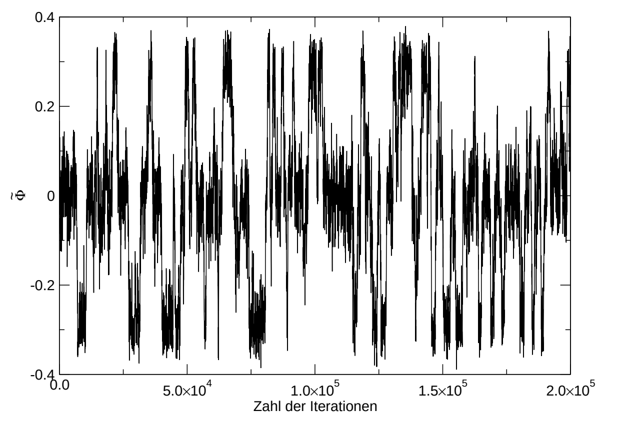
- More usual quantity:

$$\tilde{\Phi} = \frac{1}{L^3} \sum_{\vec{x}} \Phi(\vec{x})$$

Finite temperature

Finite V: Tunneling
between $\tilde{\Phi} > 0$ and $\tilde{\Phi} < 0$
$\Rightarrow$ always $\langle \tilde{\Phi} \rangle = 0$



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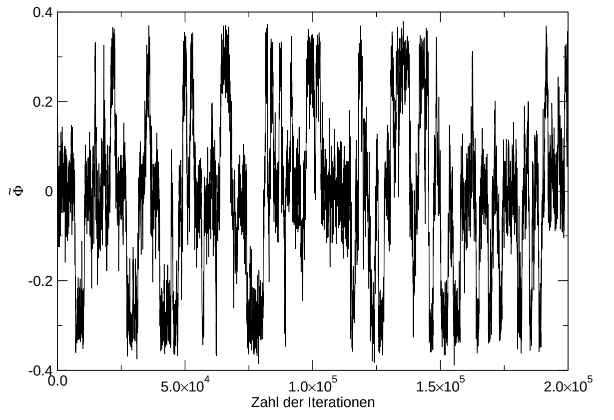

Finite V: Tunneling
between $\tilde{\Phi} > 0$ and $\tilde{\Phi} < 0$
$\Rightarrow$ always $\langle\tilde{\Phi}\rangle = 0$

$V \to \infty$:

- Tunneling suppressed
- $\Rightarrow \tilde{\Phi} = \frac{1}{L^3}\sum_{\vec{x}} \Phi(\vec{x})$ serves as order parameter for confinement:

$\langle\tilde{\Phi}\rangle \neq 0 \Leftrightarrow$ center symmetry spontaneously broken

cp. $|\langle\tilde{\Phi}\rangle| \sim \exp\left(-\hat{F}_Q N_\tau\right)$

Finite temperature

Important quantities:

- Susceptibility $\chi = \left\langle \Phi^2 \right\rangle - \left\langle \Phi \right\rangle^2$

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- Susceptibility $\chi = \left\langle \Phi^2 \right\rangle - \left\langle \Phi \right\rangle^2$
- Binder cumulant: $g_R \equiv \frac{\langle \Phi^4 \rangle}{\langle \Phi^2 \rangle^2} - 3$
  - 2^{nd} order PT $\Rightarrow g_R^c = -2$
  - 4^{th} cumulant of $\tilde{\Phi}$ (i.e. order parameter in *in*finite volume)

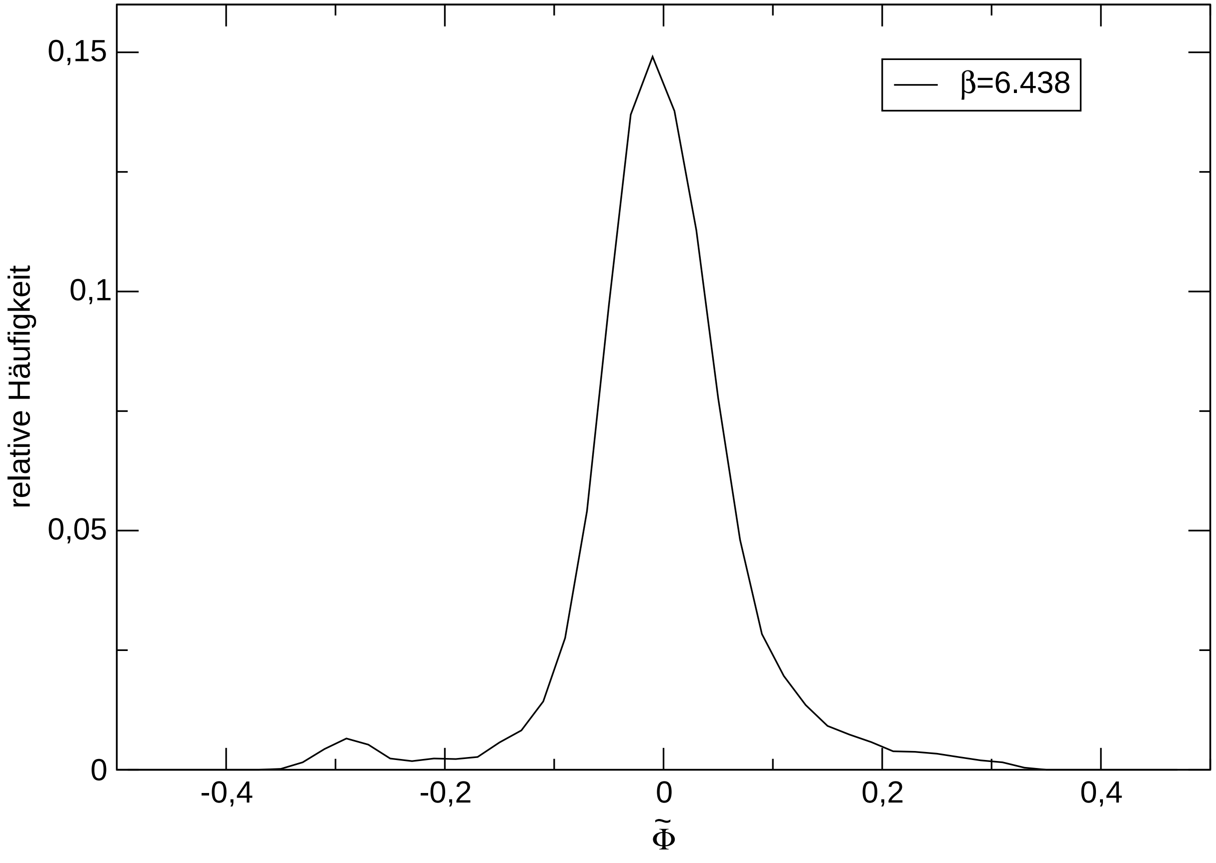
Finite temperature

Phase transition: Results ($N_\tau = 2$)

- 1st order PT at $\beta = 6.443(2)$

  (Holland, Pepe, Wiese NPB (PS) 129 ('04): $\beta = 6.4643(2)$)

Finite temperature



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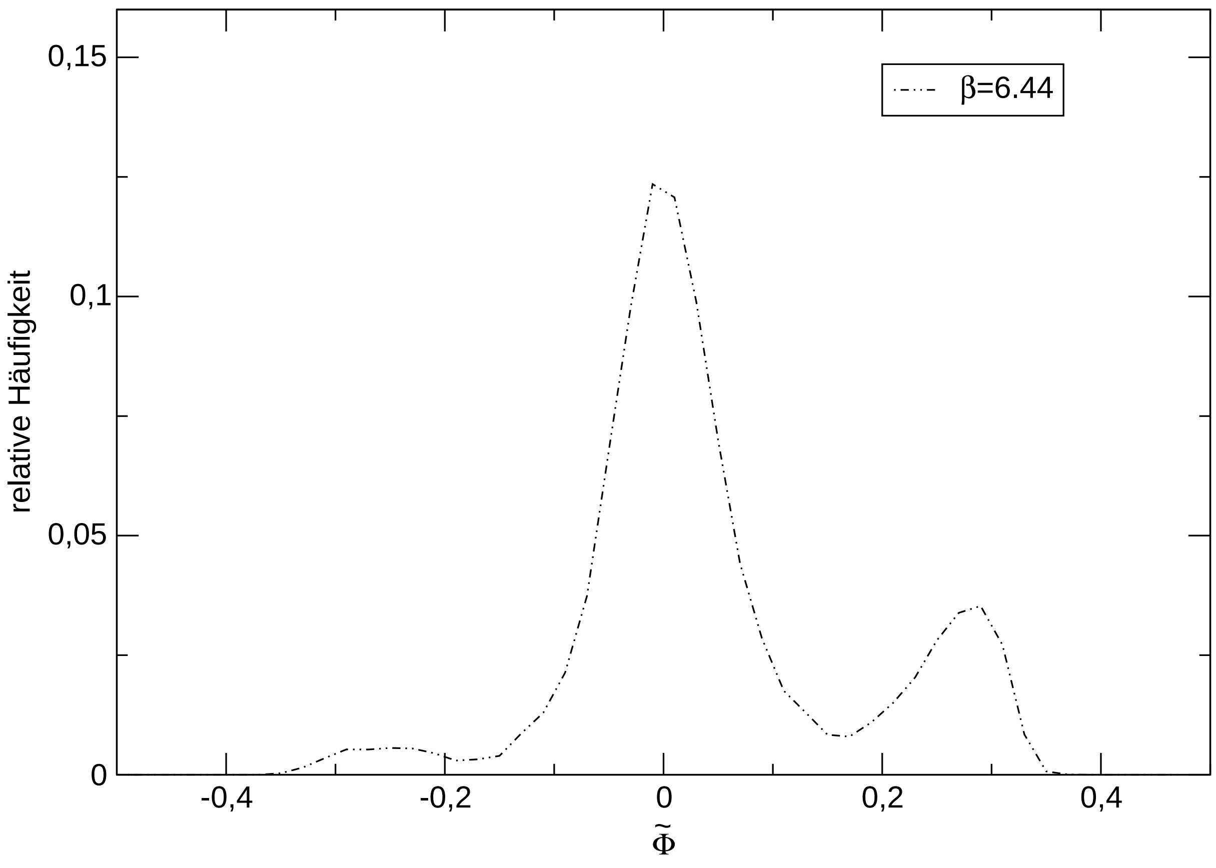

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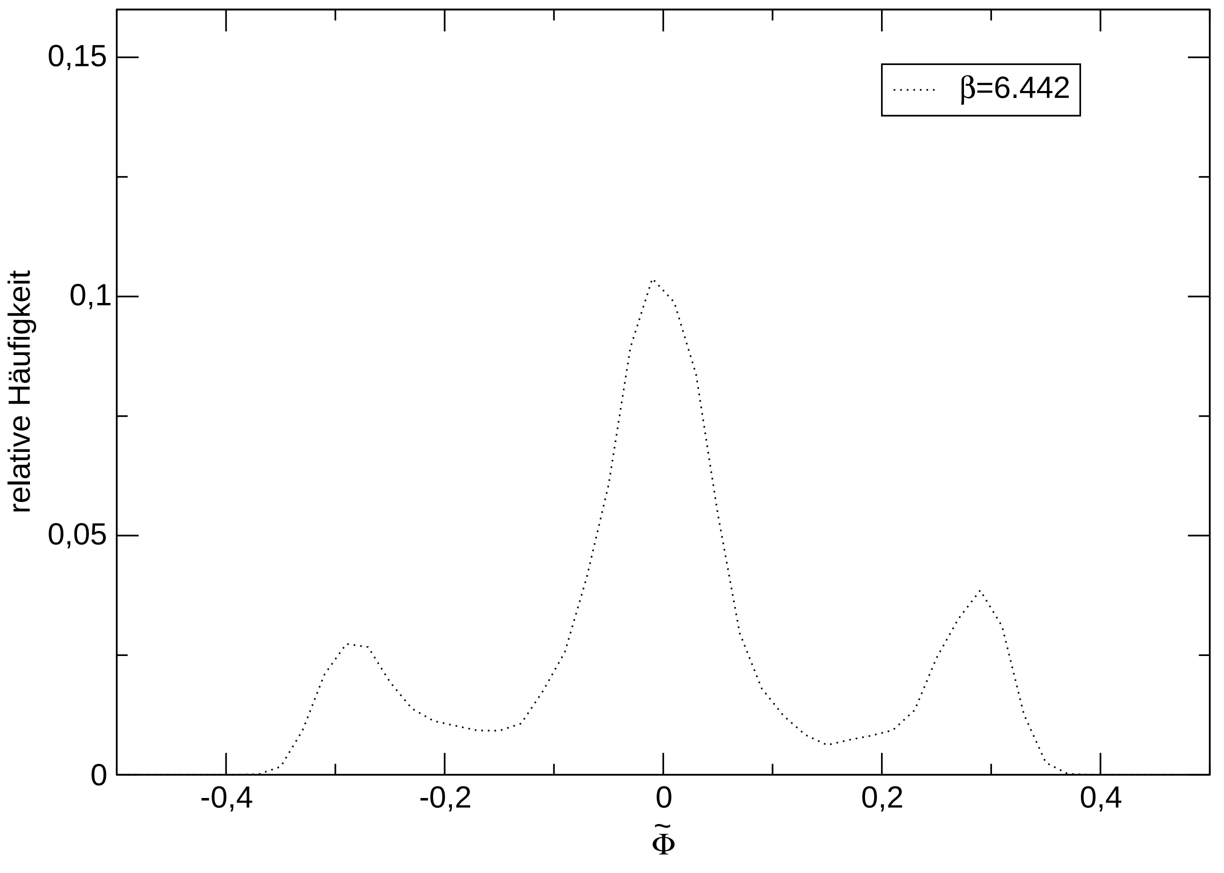

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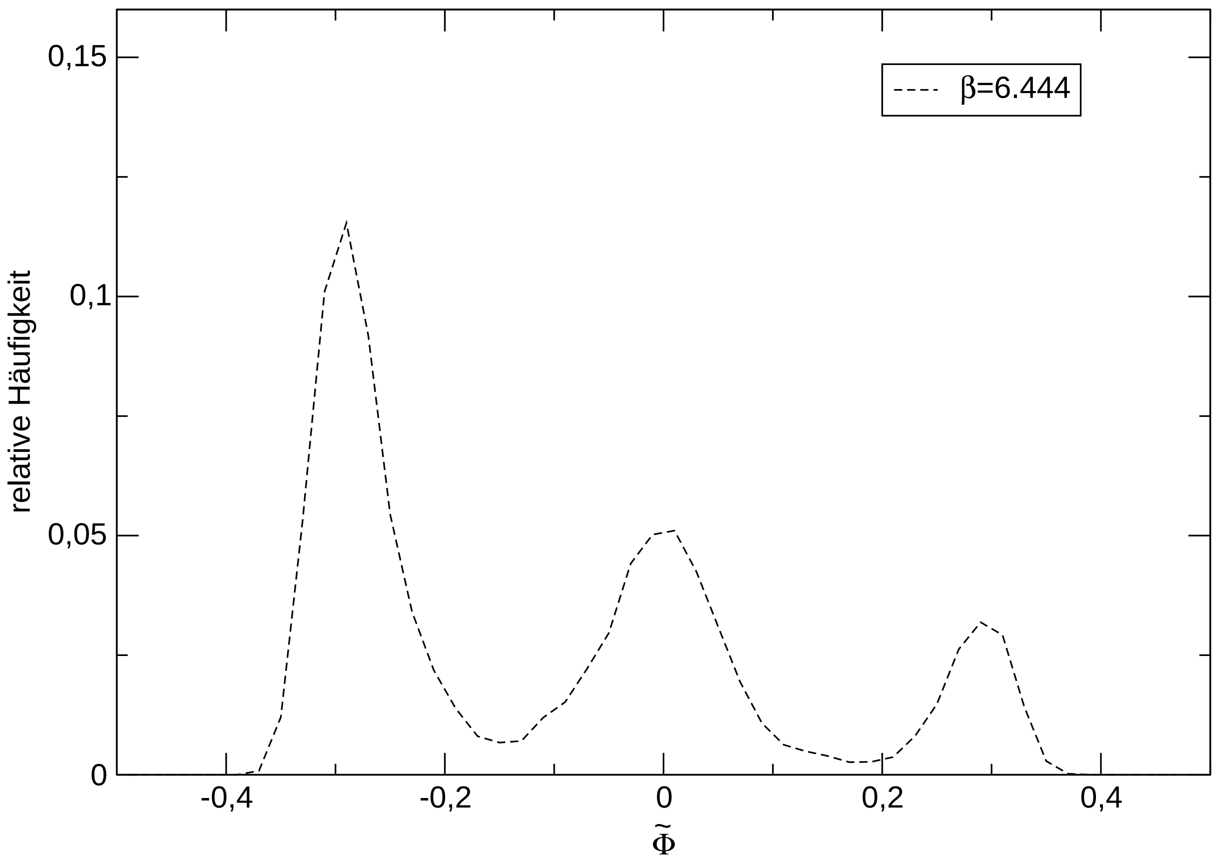

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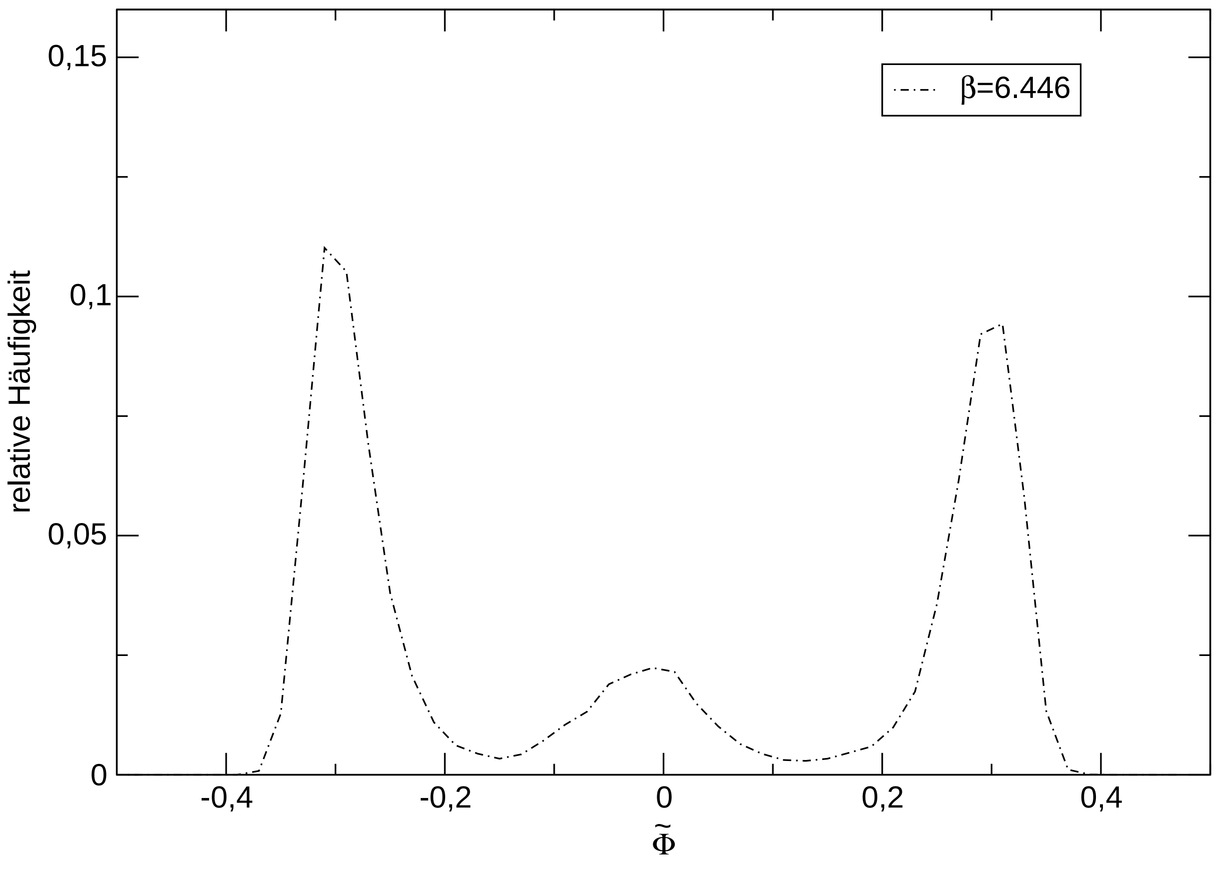

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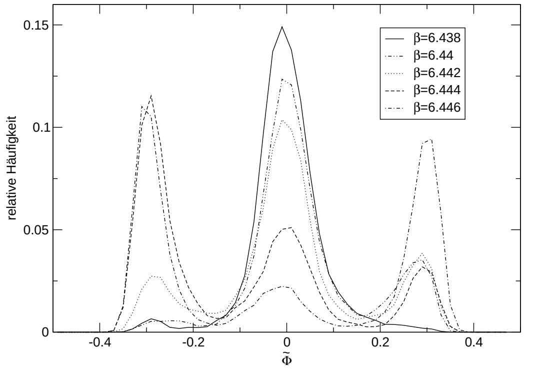

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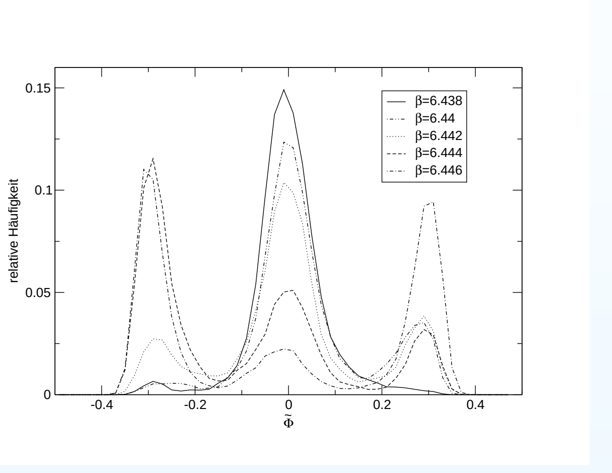

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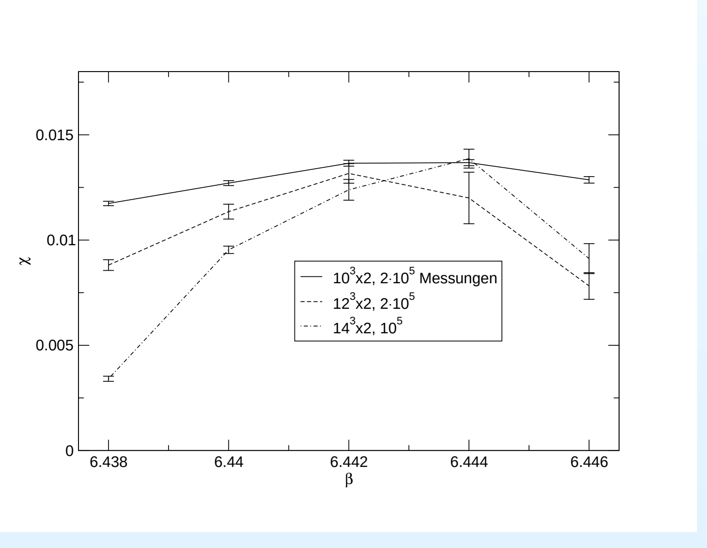
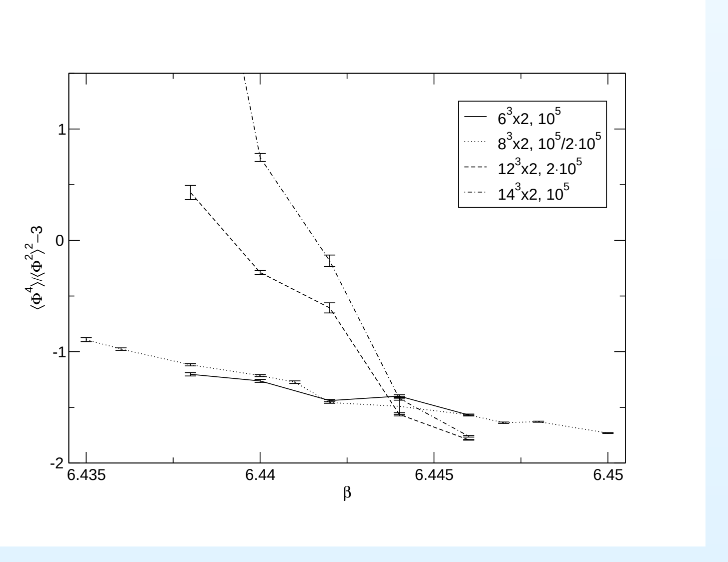
- Confirmed by χ and g_R data:





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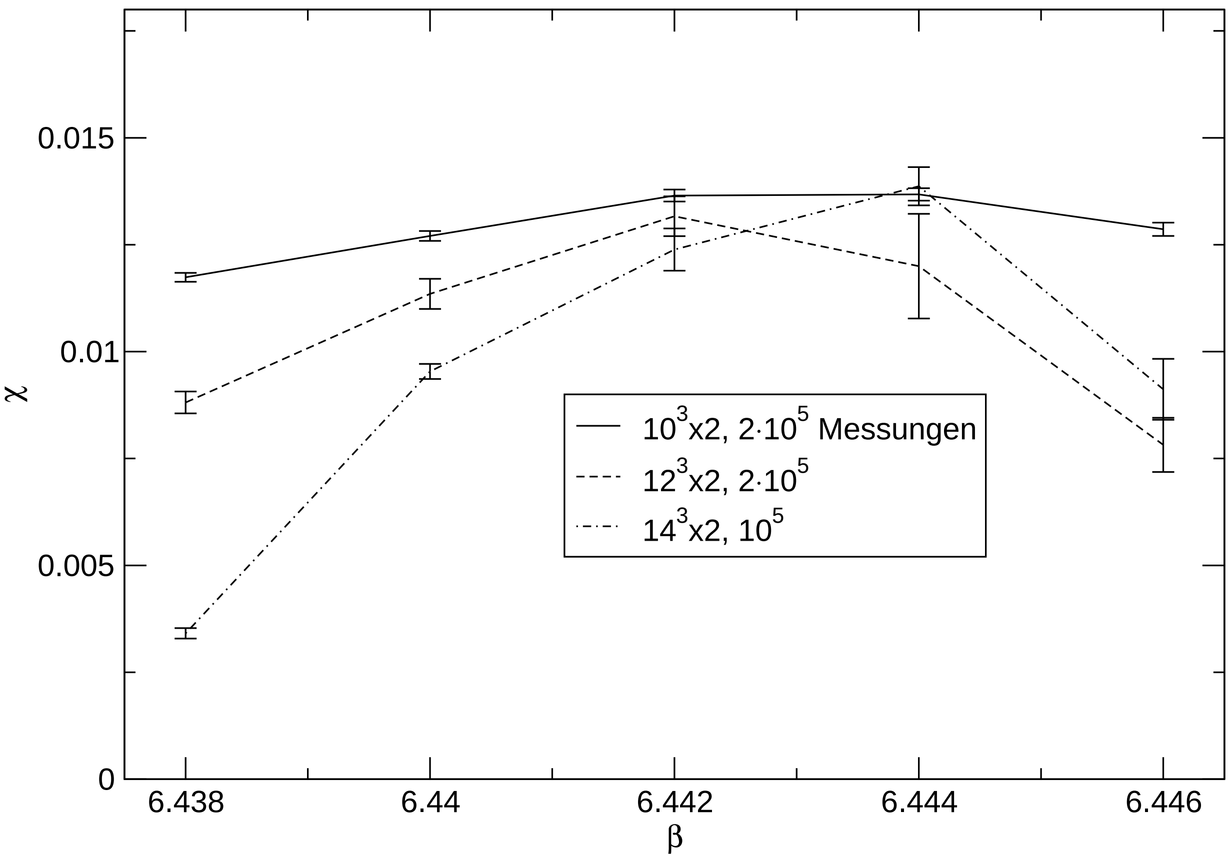

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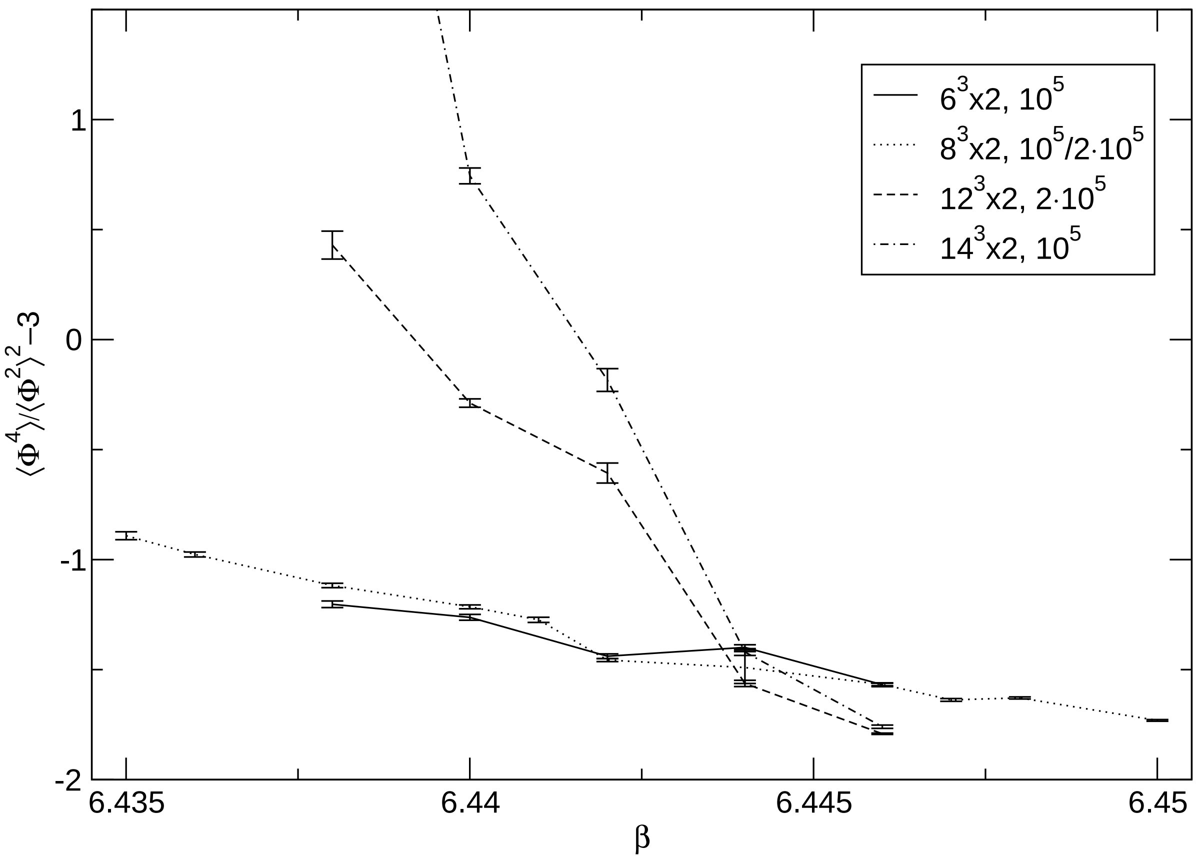

Finite temperature

Polyakov loop holonomy near the phase transition

- GT of a link variable: $U_\mu(x) \mapsto \Omega(x)U_\mu(x)\Omega^\dagger(x+\hat{\mu})$

Finite temperature

Polyakov loop holonomy near the phase transition

- GTs $P(\vec{x}) \mapsto \Omega(\vec{x},0)P(\vec{x})\Omega^\dagger(\vec{x},0)$
  are conjugations (valid for any closed contour)
- $\Rightarrow$ Local GT of $P(\vec{x})$ into Cartan subgroup always possible
  (Blau, Thompson Commun. Math. Phys. 171 ('95))
  ($H \ni g =$ diag $(\exp(i\alpha_3), \exp(-i\alpha_3), \exp(i\alpha_6), \exp(-i\alpha_6))$)

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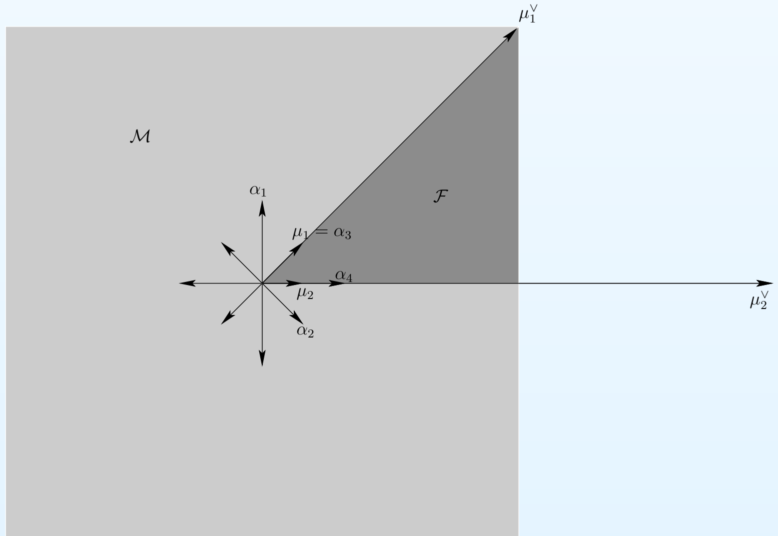
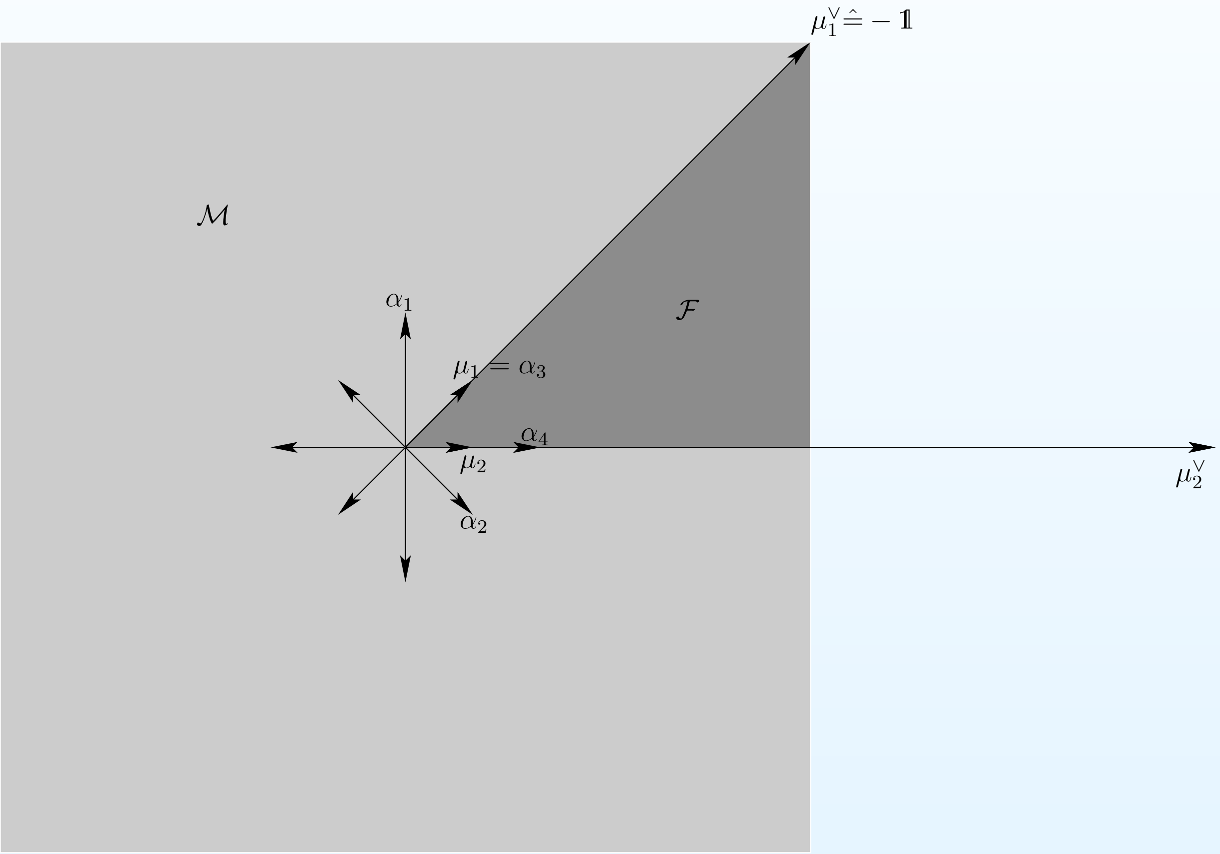


- In case of $Sp(2)$:
  Reflections at roots
  (here: Weyl group W)
  are GTs

Finite temperature



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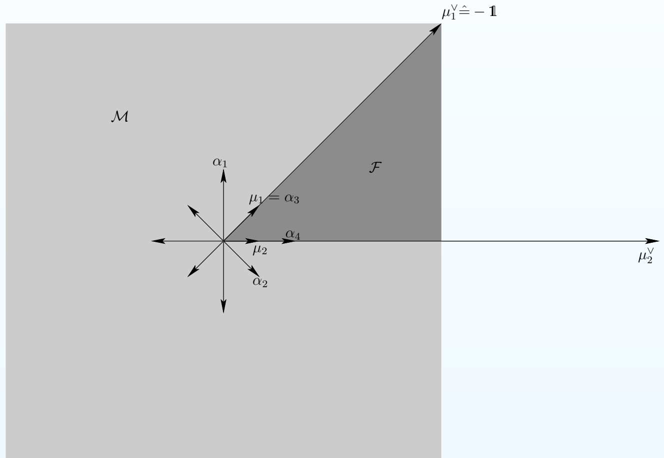
Polyakov loop holonomy near the phase transition

- $\Rightarrow$ GT maps P to a definite Weyl chamber $\mathcal{H}/W$. Constraint to bijective region yields fundamental domain $\mathcal{F} = \mathcal{H}/W \cap \mathcal{M}$.



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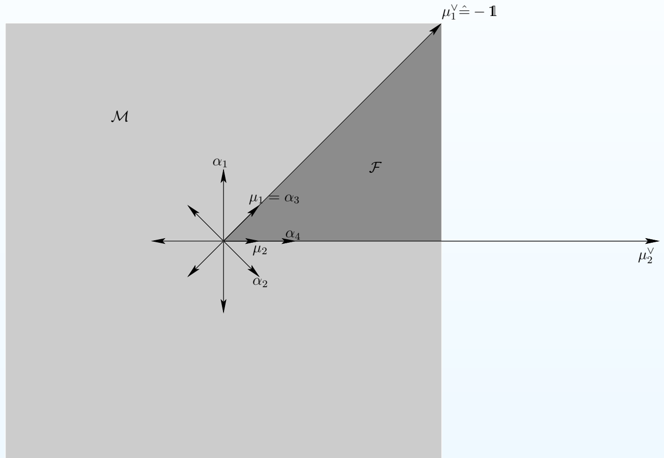

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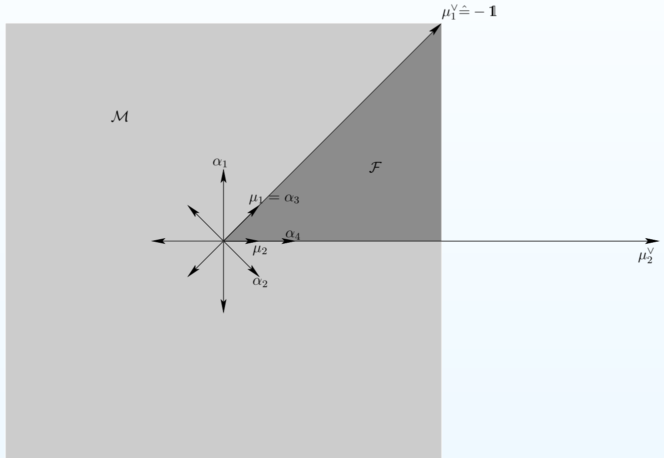

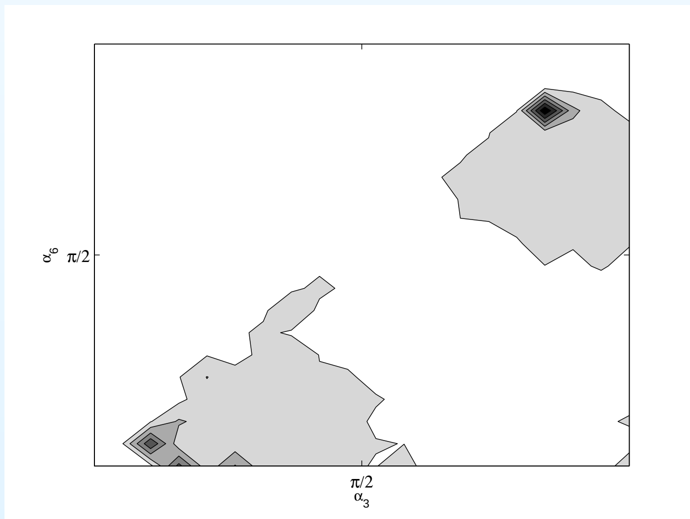

Results:

$$\beta = 6.438$$
(slightly below PT)

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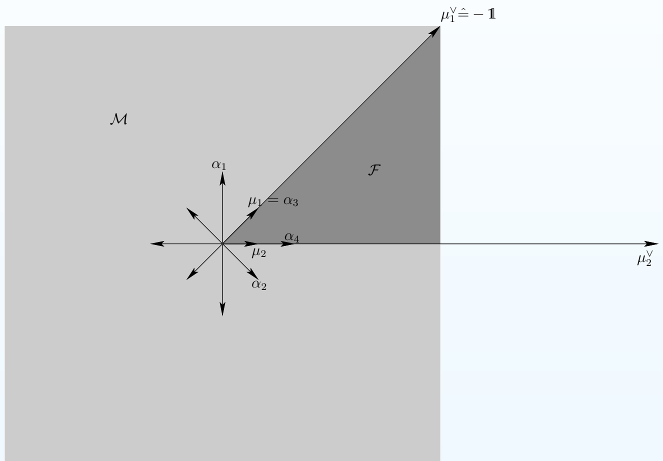
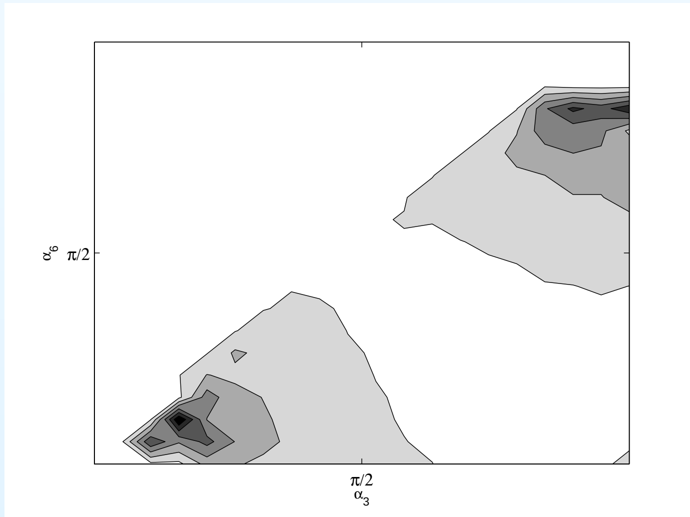


Results:

$$\beta = 6.446$$

(slightly above PT)



Subgroups at $T \approx 0$: Method

General remarks:

- Partial gauge fixing necessary before projecting to subgroup

  (Faber, Greensite, Olejník JHEP 9901 for center and Abelian proj. of $SU(2)$)

  Algorithms:

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- Calculate $\sigma a^2(\beta)$ of projected and 'reduced' config's

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Candidate subgroups:

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- $\mathcal{Z}(Sp(2)) = \mathbb{Z}_2$
  - Direct center gauge

Subgroups at $T \approx 0$: Method

Candidate subgroups:

- Diagonal $SU(2)$: $\begin{pmatrix} g & 0 \\ 0 & g \end{pmatrix}$, $g \in SU(2)$
- $U(1) \times U(1)$: diag $(\exp(i\alpha_3), \exp(-i\alpha_3), \exp(i\alpha_6), \exp(-i\alpha_6))$
  Maximal Abelian Subgroup
- $U(1)$
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'Ideal gauge fixing' (LANGFELD PRD 69 ('04))

- Aim: Minimize distance of subgroup elements $V_\mu(x)$ from gauge-transformed full gauge field $U_\mu^\Omega(x)$

$$\sum_{x,\mu} \left\| U_\mu^\Omega(x) - V_\mu(x) \right\|^2 \xrightarrow{\Omega, V_\mu} \text{min}$$

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$$\sum_{x,\mu} \left\| U_\mu^\Omega(x) - V_\mu(x) \right\|^2 \xrightarrow{\Omega, V_\mu} \text{min}$$

$\Longleftrightarrow$ Maximize the quantity

$$R \equiv \frac{1}{4NL^4} \sum_{x,\mu} \text{Re}\,\text{tr}\left(U_\mu^\Omega(x) V_\mu^\dagger(x) \right) \in [0,1]$$

by adjusting $\{V_\mu(x)\}$ and $\{\Omega(x)\}$ simultaneously

Subgroups at $T \approx 0$: Method

'Ideal gauge fixing'

- Algorithm finds mere *local* maxima of R
  $\Rightarrow$ Gribov problem on the lattice
  - Gribov region: *local* maxima of the gff
  - Fundamental modular region: *global* maxima

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'Ideal gauge fixing'

- Algorithm finds mere *local* maxima of R
  $\Rightarrow$ Gribov problem on the lattice
  - Gribov region: *local* maxima of the gff
  - Fundamental modular region: *global* maxima
- Decomposition: $U = UV^\dagger V \equiv \tilde{U} V$
  ($\not\Rightarrow \langle W \rangle = \langle \tilde{W} \rangle \langle W_V \rangle$)
  - Projected config's: $\{V_\mu(x)\}$
    Hope: $\sigma_{\text{proj}} a^2(\beta) \approx \sigma a^2(\beta)$
  - Reduced (removed) config's: $\left\{U_\mu^\Omega(x) V_\mu^\dagger(x)\right\}$
    Hope: $\sigma_{\text{r}} \approx 0$

Subgroups at $T \approx 0$: Method

Implementation of ‘ideal gauge fixing’ condition:

- Steepest ascent.
    - (Caveat: G.f. more sensitive than Smearing.)

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  - $\Omega(x) \to \exp\left(\sum_a \vartheta^a C^a\right)\Omega(x)$
  - Local change of gff:
    $\tilde{r}' \approx \underbrace{\mathrm{Re\,tr}\left(\Omega M\right)}_{\tilde{r}} + \vartheta^a \mathrm{Re\,tr}\left(C^a \Omega M\right)$
    where $M = \sum_{\nu=0}^{7} U_\nu(x)\Omega^\dagger(x+\hat{\nu})V_\nu^\dagger(x)$

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- $\vartheta^a = \varepsilon_{\text{gf}} \text{Re tr}\left(C^a \Omega M\right)$
  $\Rightarrow \Delta\tilde{r} > 0$ (1^{st} order)

Subgroups at $T \approx 0$: Method

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- Steepest ascent.
  - (Caveat: G.f. more sensitive than Smearing.)

  Method:
  - Optimal choice of $V_\mu(x)$
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  Comments:
  - Sufficient for $Sp(2) \to SU(2)$ ($10 \to 3$ continuous d.o.f.).
  - In case of smaller subgroups: Algorithm gets caught in mere local maxima too easily.

Subgroups at $T \approx 0$: Method

Implementation of ‘ideal gauge fixing’ condition:

- Simulated annealing.

Subgroups at $T \approx 0$: Method

Implementation of ‘ideal gauge fixing’ condition:

- Simulated annealing.
  Method:
  - Random $V_\mu(x)$ (e. g. $\in \{\pm\mathbb{1}\}$) accepted with

    $$p = \text{min}\left(1, \exp\left(\frac{\beta_F}{4}\Delta\tilde{R}\right)\right)$$

    $\Rightarrow$ Allows downhill steps ($\leadsto$ leaving local maxima)
    Slow increase of $\beta_F \hat{=}$ cooling
    ($\beta_F \longrightarrow \infty \hat{=}$ steepest ascent)
  - Update of $\Omega(x)$ as before

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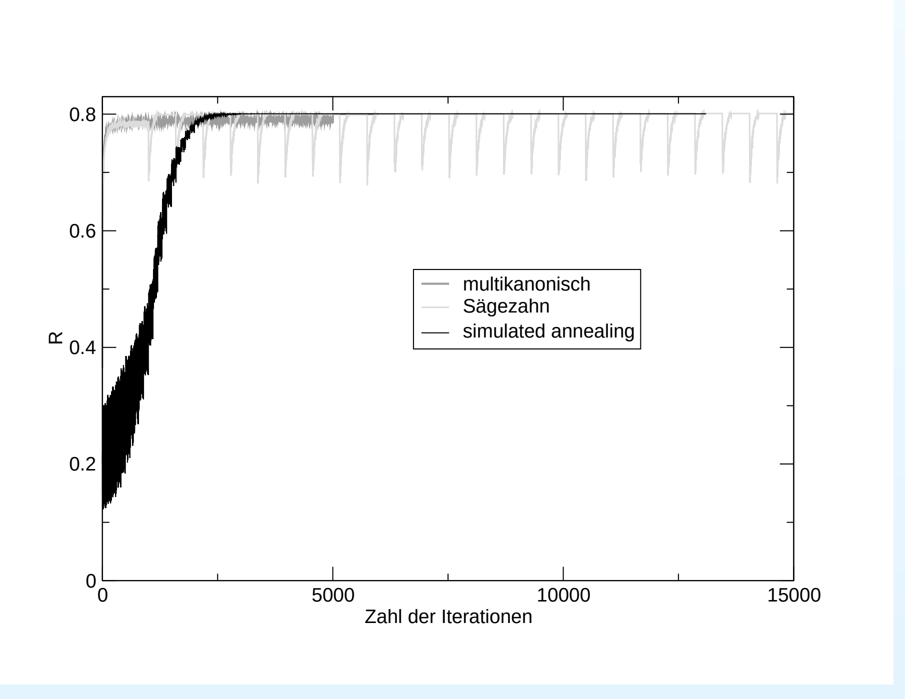
  Comments:
  - Much more time-consuming than steepest ascent
  - Suitable method for center projection

Subgroups at $T \approx 0$: Method

Implementation of ‘ideal gauge fixing’ condition:

- Simulated annealing
  Comparison with other (experimental) algorithms:
  - Raising & lowering β_F several times
  - Lowering β_F when approaching a local maximum



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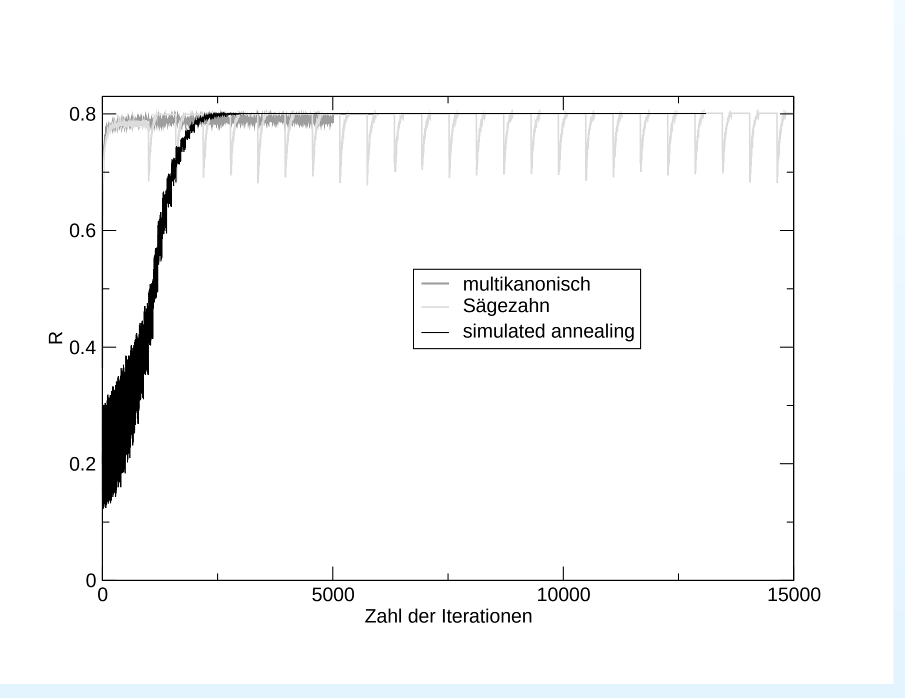

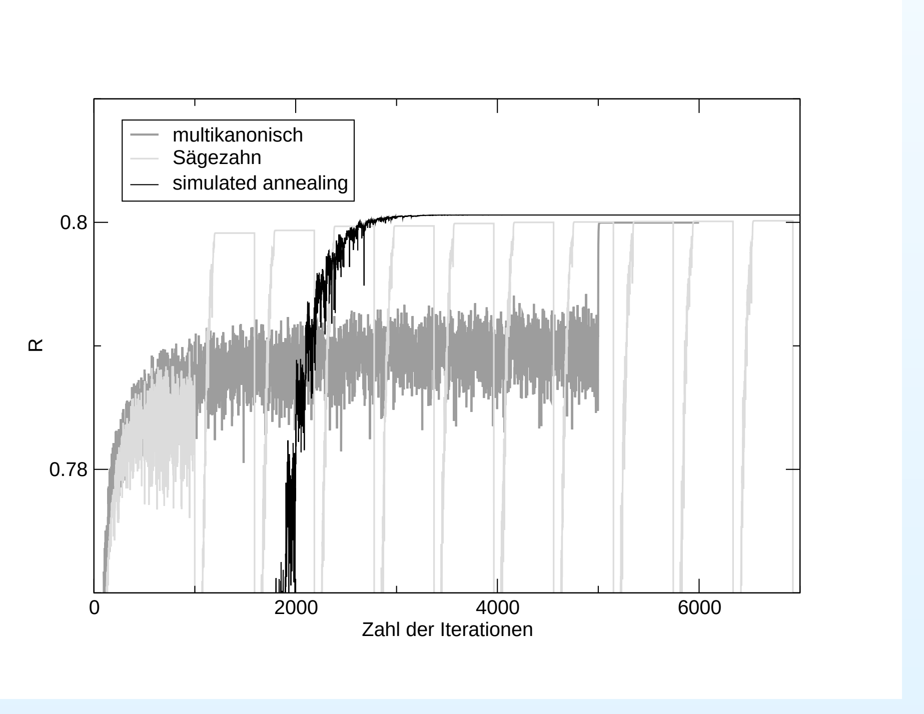

Subgroups at $T \approx 0$: Method

Implementation of 'ideal gauge fixing' condition:

- Abelian projection (to $U(1)$ and $U(1) \times U(1)$):
  Small, but continuous subgroups. MC for V less obvious.
  Steepest ascent also unsatisfactory. – Method:
  - Optimal choice of $V_\mu(x)$.
  - Simulated annealing for $\Omega(x)$.
    - $p = \exp\left(\frac{\beta_F}{4}\Delta S\right)$, increase β_F
    - Adjust ε_{gf} s. t. acc. rate for new $\Omega(x) \approx 50\%$

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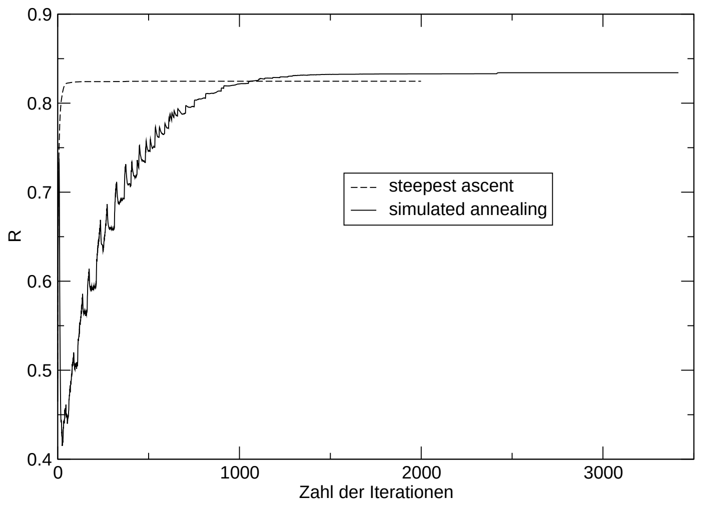

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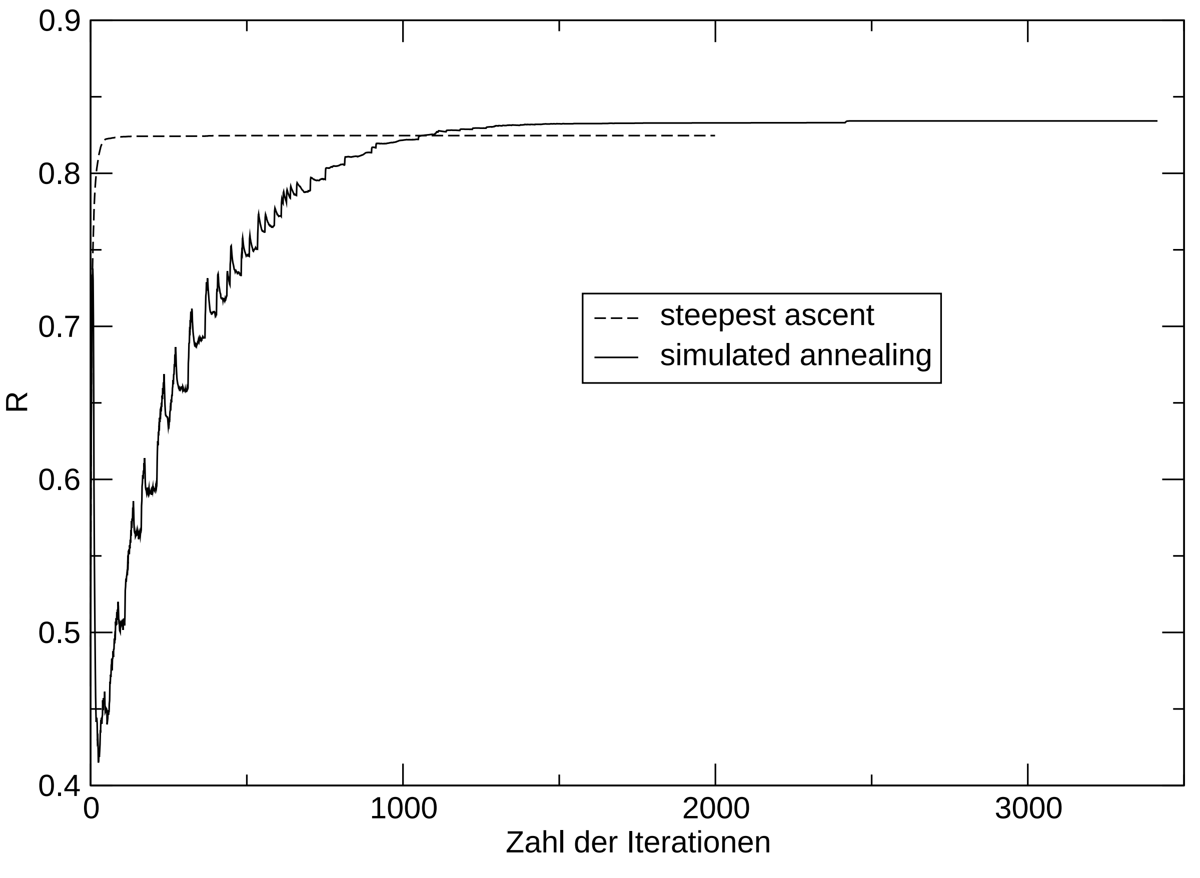
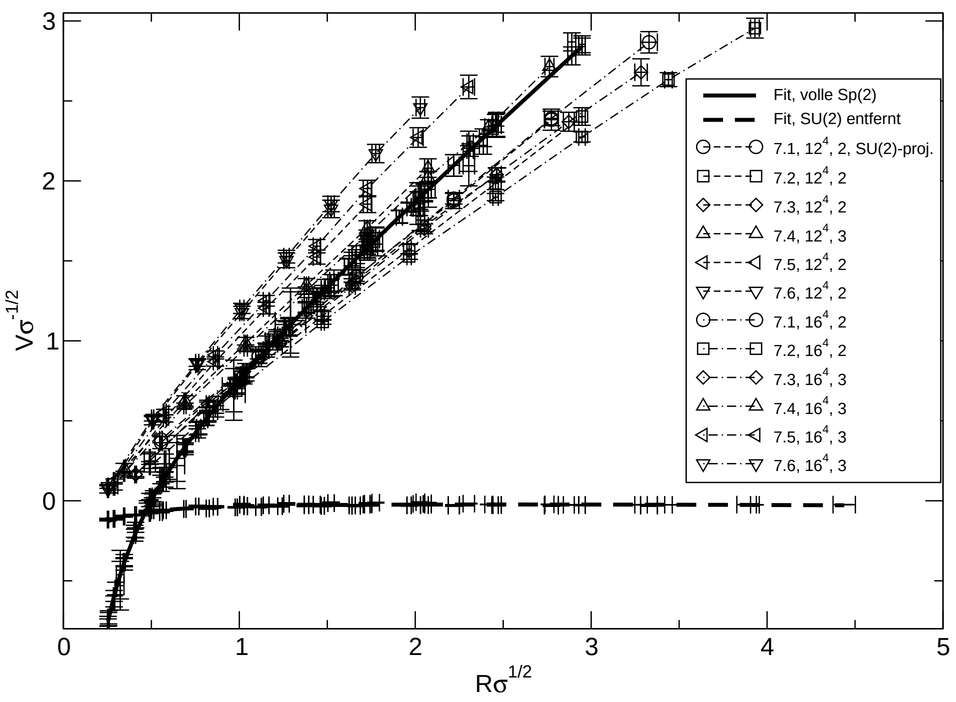


Subgroups at $T \approx 0$: Results

Results for $SU(2)$

- Sufficient for confinement: $\sigma_{\text{proj}} \approx \sigma$
- Necessary for confinement: $\sigma_{\text{r}} \approx 0$



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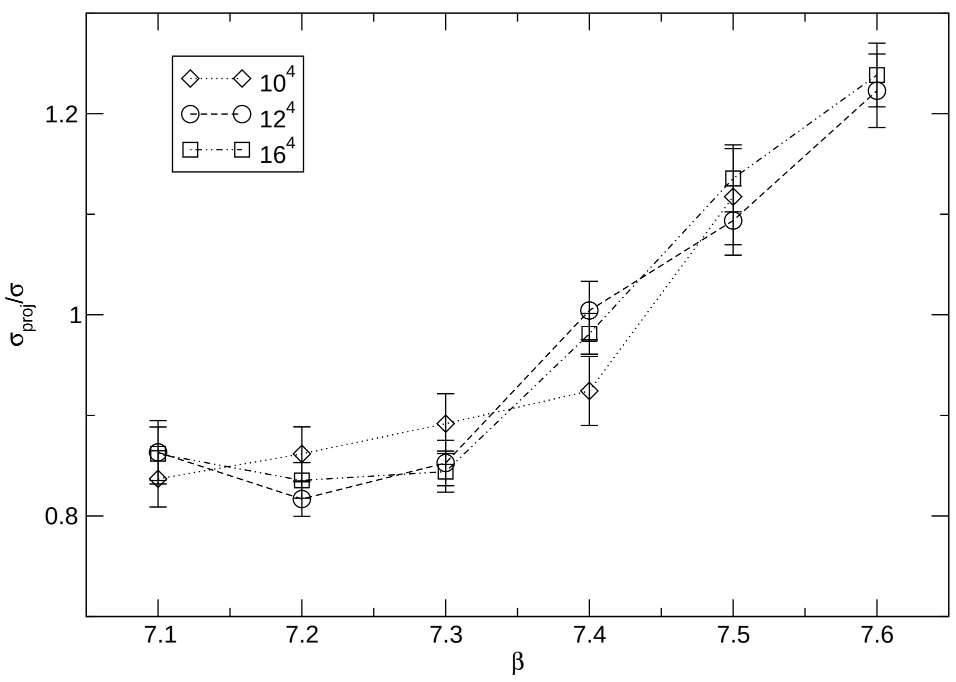

Subgroups at $T \approx 0$: Results

Results for $SU(2)$

- Evidence for nontriviality of '$SU(2)$ dominance':
  - $\sigma_{\text{proj}}/\sigma$ increases as $a \to 0$
  - Coulomb coefficient drops by 75% after partial g.f. and projection

$$\text{(remember: } V = \sigma R - \frac{\alpha}{R} + c\text{)}$$

  - (Phase transition after projection at same temperature)

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  - Coulomb coefficient drops by 75% after partial g.f. and projection
  - (Phase transition after projection at same temperature)
- Problems:
  - $SU(2)$ is too large a subgroup to be really interesting
  - No direct identification of topological objects

Subgroups at $T \approx 0$: Results

Previous results on Abelian dominance:

- Abelian projection proposed by Mandelstam & 't Hooft
- Connected to dual Meissner effect as hypothetical mechanism of confinement

Subgroups at $T \approx 0$: Results

Previous results on Abelian dominance:

- Abelian projection proposed by Mandelstam & 't Hooft
- Connected to dual Meissner effect as hypothetical mechanism of confinement
- Numerical corroboration of Abelian dominance in $SU(2)$: $\sigma_{\text{proj}} > 0.9\sigma$ (Hioki et al. PLB 272 ('91); Ilgenfritz et al. PRD 61 ('00))
- $SU(3)$: $\sigma_{\text{proj}} \approx 0.8\sigma$ (Stack, Tucker, Wensley NPB 639 ('02))
  - Caveat: σ_{proj} drops when g.f. is improved

Subgroups at $T \approx 0$: Results

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- $\sigma_{\mathrm{r}} \approx 0$
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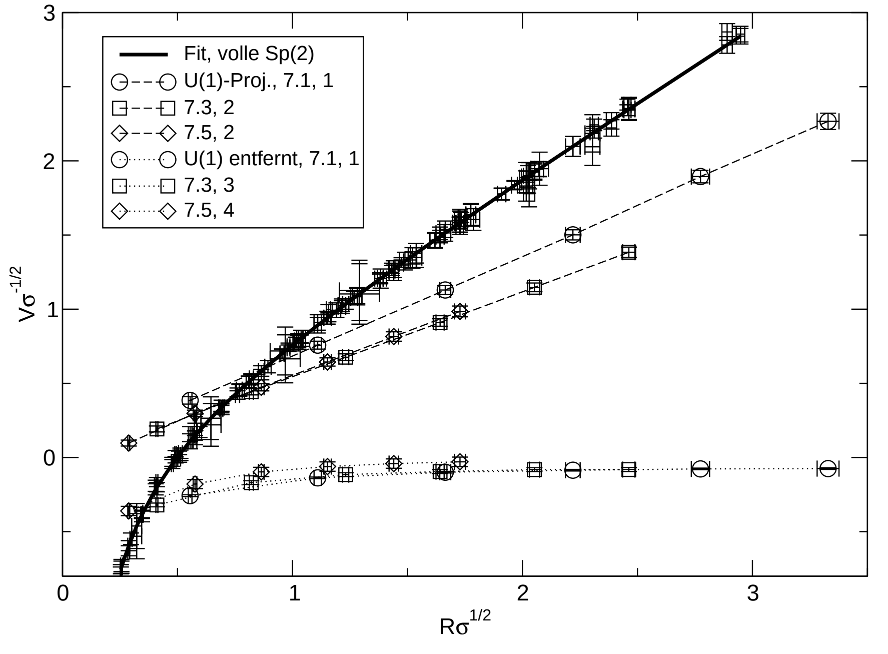

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Results for Abelian subgroups:

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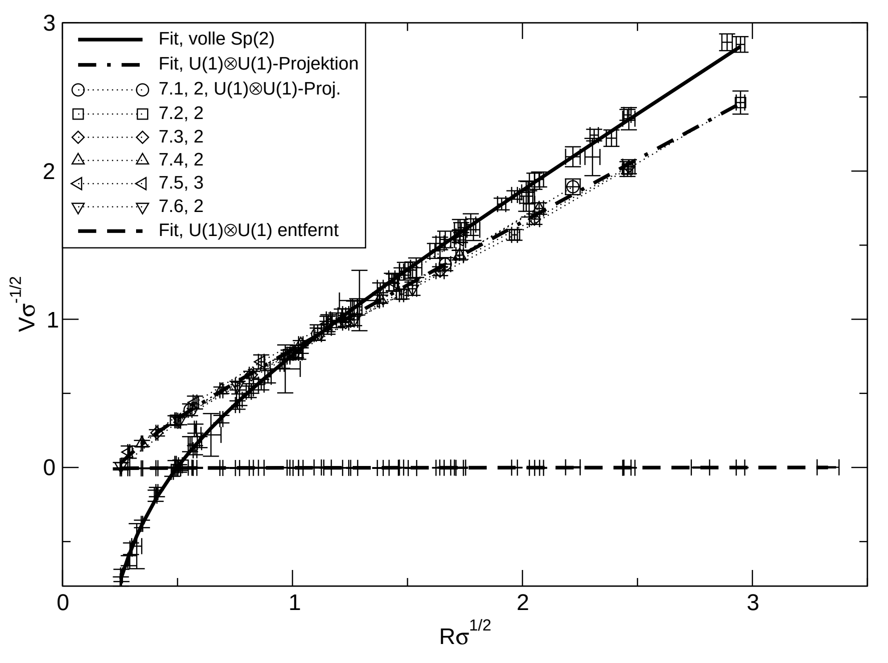

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  - 100% in LCG (DE FORCRAND, PEPE NPB 598 ('01))
  - 80% in IMCG via $U(1) \times U(1)$
    (STACK, TUCKER, WENSLEY NPB 639 ('02))

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  - 80% in IMCG via $U(1) \times U(1)$ (STACK, TUCKER, WENSLEY NPB 639 ('02))
  - 62% in MCG, 58% in ICG (LANGFELD PRD 69 ('04), KUSTERER '04)

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Center dominance in $Sp(2)$ not to the same extent as in $SU(2)$ and $SU(3)$!

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  - Caution: adequate g.f. proves difficult

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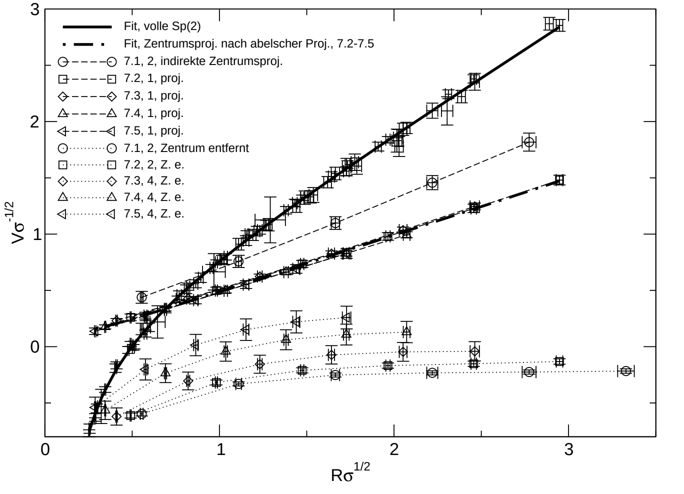
- Indirect ICG via $SU(2)$: $\sigma_{\text{proj}} \approx 0.43\sigma$
- IICG via $U(1) \times U(1)$: $\sigma_{\text{proj}} \approx 0.49\sigma$



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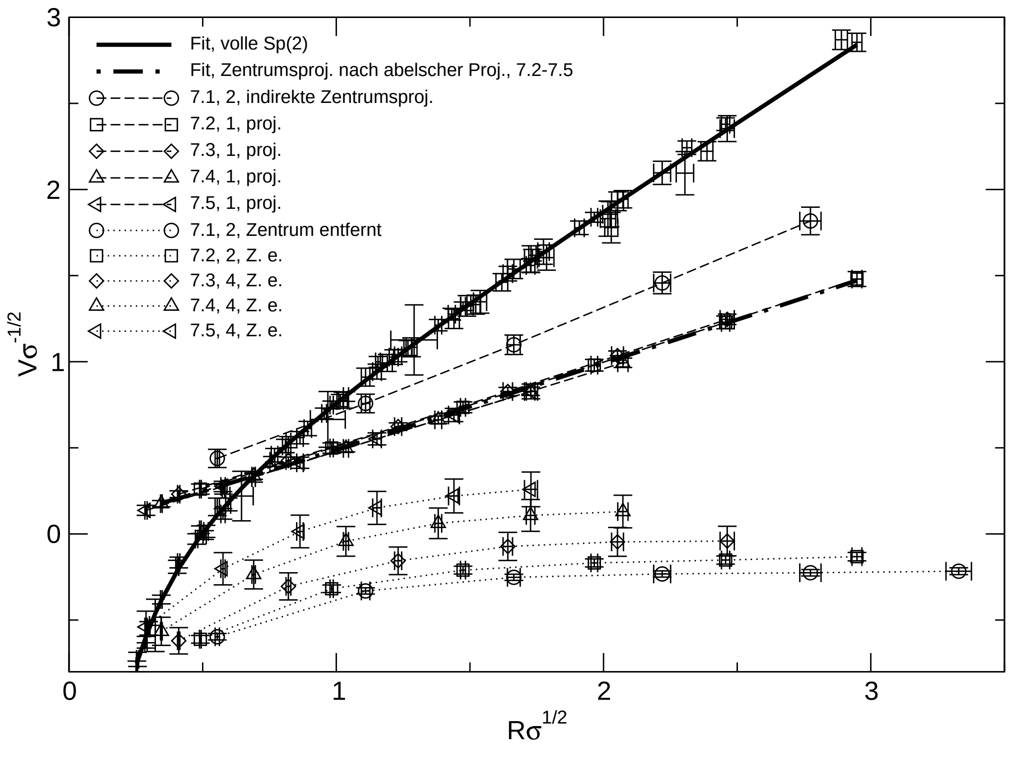

Subgroups at $T \approx 0$: Results

Topological objects that may account for confinement:

- Abelian monopoles (dual superconductor picture of confinement) (NAMBU, MANDELSTAM, 'T HOOFT)
- Center vortices ('T HOOFT, NIELSEN, AMBJØRN, OLESEN)

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Previous results:
Monopoles:

- $SU(2)$ MAG: $\langle W_{U(1)} \rangle \approx \langle W_{\text{mon}} \rangle \langle W_{\text{ph}} \rangle$
  $\Rightarrow V_{U(1)}(r) \approx V_{\text{mon}}(r) + V_{\text{ph}}(r)$
  (Shiba, Suzuki PLB 333 ('94); Stack, Neiman, Wensley PRD 50 ('94))
- Monopole potential reproduces full σ (Bali et al. PRD 54 ('96))
- $SU(2)$ MAG: $\rho/\sigma^{3/2} = 0.65$
  (Bornyakov, Müller-Preussker NPB (PS) 106 ('02))
- Finite continuum limit of ρ in $SU(2)$ LAG doubtful (Bornyakov, Ilgenfritz, Müller-Preussker PRD 72 ('05))

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- Center vortices ('t Hooft, Nielsen, Ambjørn, Olesen)

Previous results:
Center vortices:

- Random vortex model $\Rightarrow \rho/\sigma = 0.5$
- $SU(2)$ IMCG: $\rho/\sigma = 0.6$ (Stack, Tucker, Wensley NPB 639 ('02))
  $SU(2)$ LCG: ρ/σ divergent (Langfeld, Reinhardt, Schäfke PLB 504 ('01))
- $SU(3)$ MCG: $\rho/\sigma = 0.45$ (Langfeld PRD 69 ('04))

Subgroups at $T \approx 0$: Results

Abelian monopoles in $U(1) \times U(1)$-projection of $Sp(2)$:

- $\pi_1(Sp(2)) = 0$ (no mp.), but $\pi_1(U(1) \times U(1)) = \mathbb{Z}^2$
- Decompose the ‘plaquette holonomy’ $U_{\mu\nu} = \exp(i\Theta_{\mu\nu})$ after projection to $U(1)$:

$$\Theta_{\mu\nu} = \underbrace{\tilde{\Theta}_{\mu\nu}}_{\in]-\pi,\pi]} + 2\pi n_{\mu\nu}.$$

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$$\rho a^3 = \frac{1}{4L^4} \sum_{\mathrm{C}} |M|$$

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$$\rho a^3 = \frac{1}{4L^4} \sum_{\mathrm{C}} \left| \varepsilon^{\mu\nu\lambda} \nabla_\mu n_{\nu\lambda} \right|$$

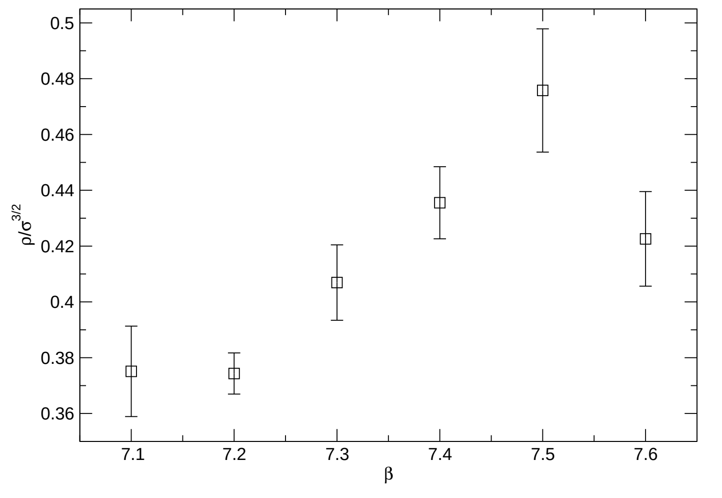
(DeGrand, Toussaint PRD 22 ('80))

Subgroups at $T \approx 0$: Results

Abelian monopoles in $U(1) \times U(1)$-projection of $Sp(2)$:

$$\lim_{a \to 0} \frac{\rho a^3}{(\sigma a^2)^{3/2}} \approx 0.44$$

$\Rightarrow$ Physical relevance *possible*



Subgroups at $T \approx 0$: Results

Center vortices in center projection of $Sp(2)$

- Random vortex model:

$$\lim_{a \to 0} \frac{\rho a^2}{\sigma a^2} = 0.5$$

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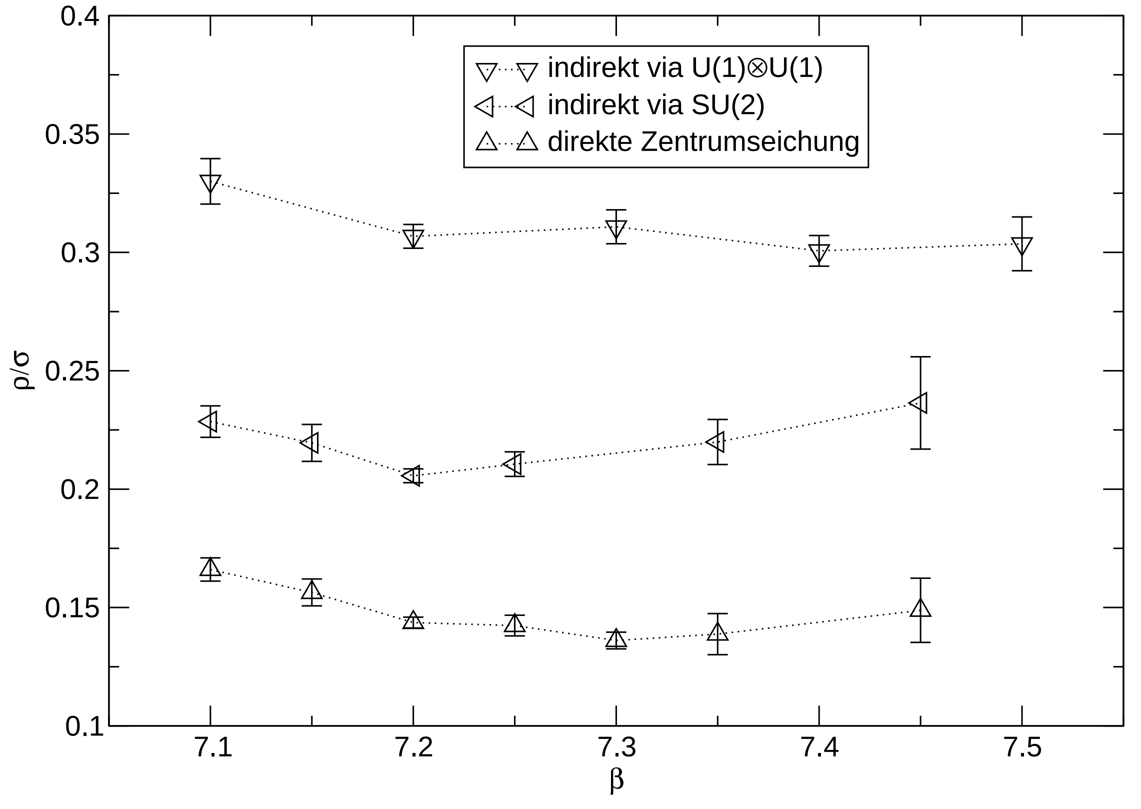
- Lattice measurement:

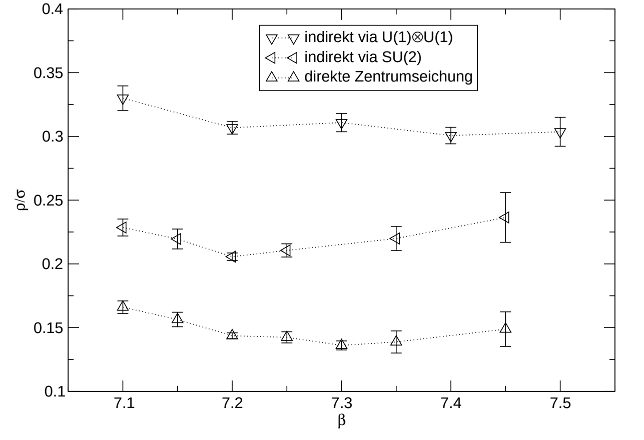


Subgroups at $T \approx 0$: Results

Center vortices in center projection of $Sp(2)$

- Lattice measurement:

$$\lim_{a \to 0} \frac{\rho a^2}{\sigma a^2} < 0.5$$



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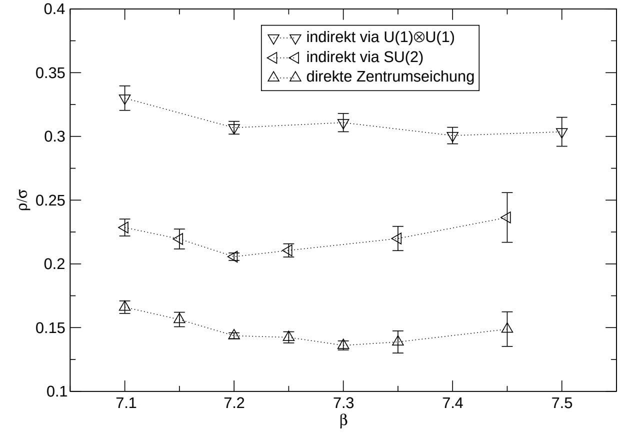
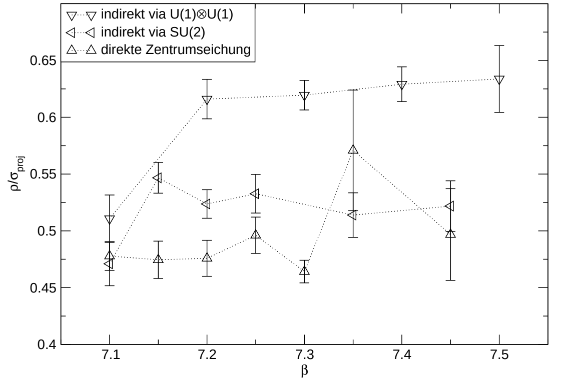


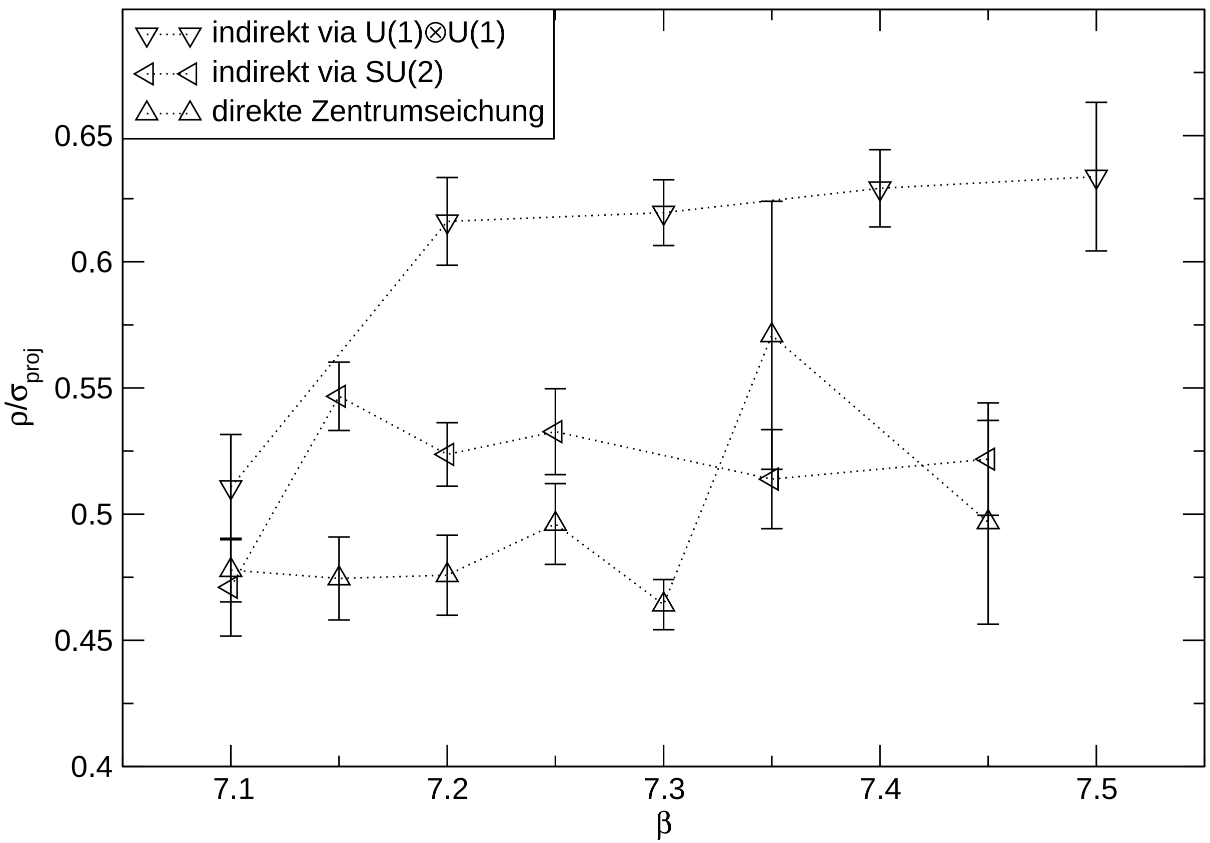
$$\lim_{a \to 0} \frac{\rho a^2}{\sigma a^2} < 0.5$$

- Instead:



$$\frac{\rho}{\sigma_{\text{proj}}} \approx 0.5 = \left(\frac{\rho}{\sigma}\right)_{\text{r. v. m.}}$$

Subgroups at $T \approx 0$: Results



Subgroups at $T \approx 0$: Results

Center vortices

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- Only if (Del Debbio et al. PRD 55 ('97))

$$\frac{W_n(\mathcal{C})}{W_0(\mathcal{C})} \xrightarrow[A(\mathcal{C})\to\infty]{} (-1)^n$$

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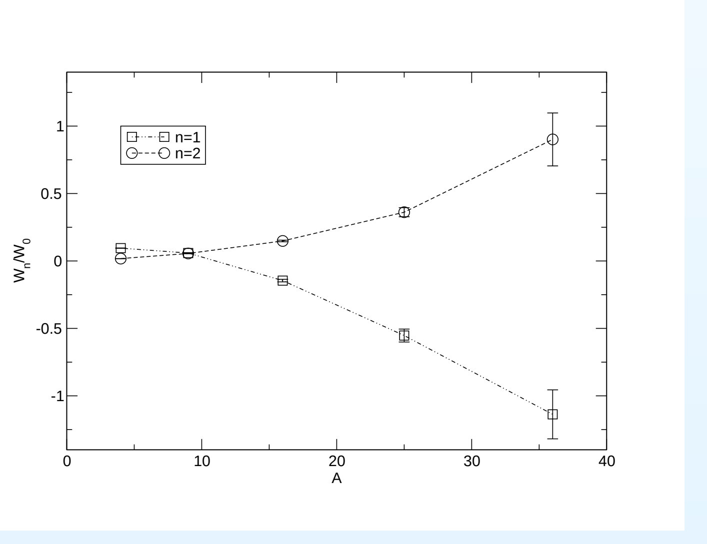
$$\frac{W_n(\mathcal{C})}{W_0(\mathcal{C})} \xrightarrow[A(\mathcal{C})\to\infty]{} (-1)^n$$

- This necessary condition is fulfilled in $Sp(2)$ IICG (via the maximal Abelian subgroup):



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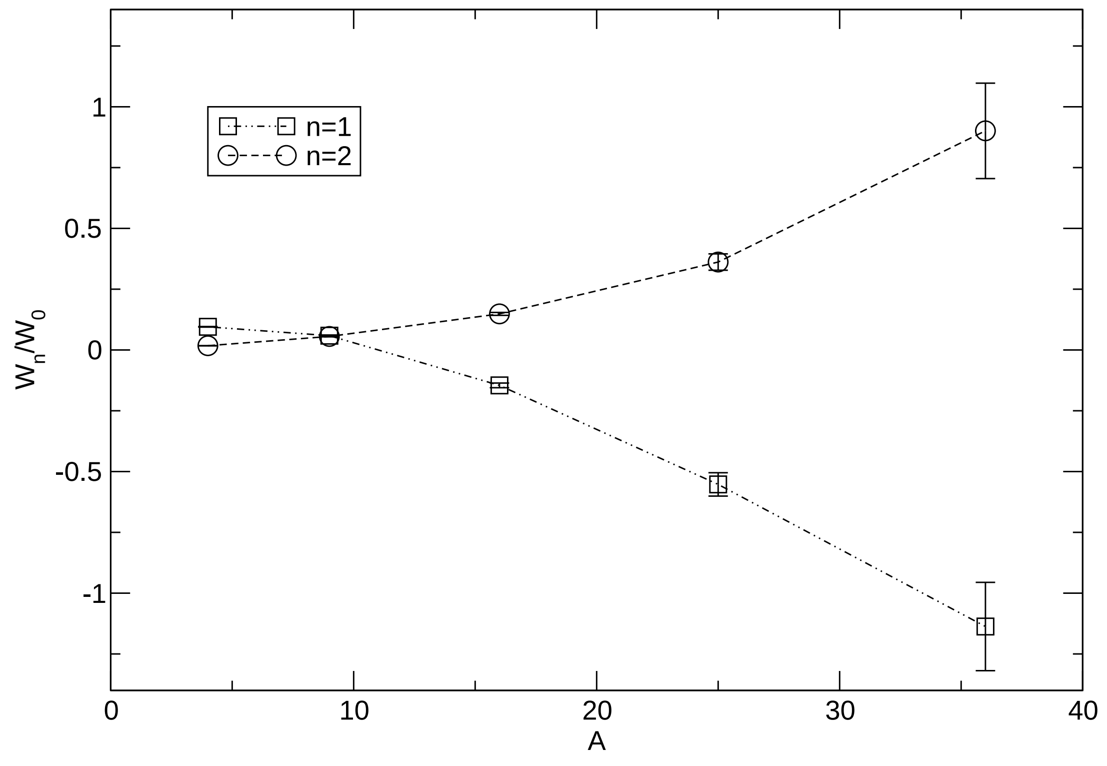
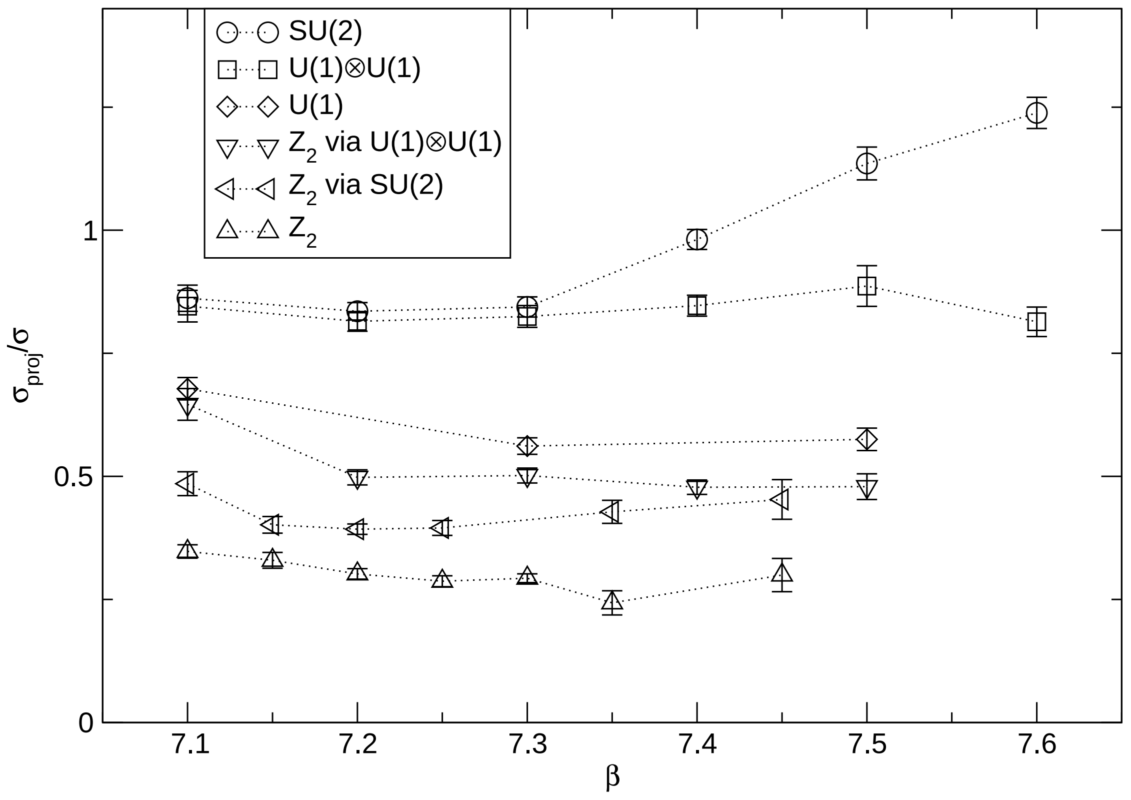


Subgroups at $T \approx 0$: Results

Summing up: $\frac{\sigma_{\text{proj}}}{\sigma}(\beta)$ after six different g.f. + projections



Conclusions

- Possibly some evidence for center relevance in $Sp(2)$ confinement mechanism from finite temperature investigations
- But: $\sigma_{\text{proj}} < 0.5\sigma$ in three center projections
- Missing center vortex density $\hat{\approx}$ missing σ_{proj}
- Maximal Abelian subgroup yields $\sigma_{\text{proj}} \approx 0.84\sigma$ (and a monopole density scaling in physical units)